MATH 162 – FALL 2004 – FIRST EXAM
SEPTEMBER 22, 2004

STUDENT NAME————————————————

STUDENT ID————————————————

RECITATION INSTRUCTOR————————————————

RECITATION TIME————————————————

INSTRUCTIONS

1. Fill in your name, your student ID number, and your recitation instructor's name and recitation time above. Write your name, your student ID number and division and section number of your recitation section on your answer sheet, and fill in the corresponding circles.
2. There are 11 questions, each worth 9 points.
3. Do all your work on the test booklet.
4. Mark the letter of your response for each question on the mark-sense sheet.
5. No books, notes or calculators may be used.

Solutions

1) Find the diameter of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z = -5$.

A) 1

B) 2

C) 3

D) 6

E) 9

$$(x^2-2x+1)+(y^2+4y+4)+z^2-6z+9) = -5+1+4+9$$

$$(x-1)^2+(y+2)^2+(z-3)^2 = 3^2$$

$$\text{diam} = 2 \cdot 3 = 6$$

2) A vector perpendicular to $\mathbf{i}+\mathbf{j}$ and $\mathbf{j}-\mathbf{k}$ is

A) $-2\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$

B) $-2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$

C) $-2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

D) $-\mathbf{i} + \mathbf{j} + \mathbf{k}$

E) $-\mathbf{i} - \mathbf{j} + \mathbf{k}$

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = i\begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} - j\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$$

$$+ k\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= -i + j + k$$

3) $\mathbf{i} \times (\mathbf{j} \times \mathbf{j})$ equals

A) $-\mathbf{k}$

B) $-\mathbf{i}$

C) $-\mathbf{j}$

D) $\mathbf{0}$

E) $\mathbf{k}$

$j \times j = 0$

$i \times 0 = 0$

4) If θ is the angle between $\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $2\mathbf{i}-\mathbf{j}$ then $\cos^2\theta$ is equal to

A) $\frac{1}{15}$

B) $\frac{1}{5}$

C) $\frac{1}{\sqrt{15}}$

D) $\frac{1}{\sqrt{5}}$

E) 1

$$\cos^2\theta = \left(\frac{(i+j-k)\cdot(2i-j)}{\|i+j-k\|\,\|2i-j\|}\right)^2$$

$$= \frac{(2-1)^2}{3\cdot 5} = \frac{1}{15}$$

5) The scalar projection of **b** $=< 1, -1, 1 >$ onto **a** $=< 2, 1, 2 >$ is

A) 1

B) $\frac{1}{9}$

C) $\frac{1}{3}$

D) $\frac{1}{\sqrt{3}}$

E) $-\frac{1}{\sqrt{3}}$

$$\frac{b \cdot a}{\|a\|} = \frac{2-1+2}{\sqrt{4+1+4}} = \frac{3}{3} = 1$$

6) The integral for the volume of the solid generated by revolving the region bounded by the curves $y = x^2$, $y = 0$, and $x = 1$ about the line $x = 1$ is

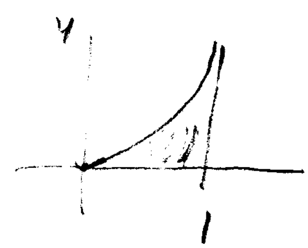
A) $\int_0^1 \pi(2x^3 - 2x^4)\, dx$

B) $\int_0^1 \pi(2x^4 - 2x^2)\, dx$

C) $\int_0^1 \pi(2x^3 - 2x^2)\, dx$

D) $\int_0^1 \pi(2x^2 - 2x^3)\, dx$

E) $\int_0^1 \pi(x^2 - x^3)\, dx$



$$\int_0^1 2\pi\, x^2(1-x)\, dx$$

7) A solid **S** has a square base in the xy-plane given by $\{0 \le x \le 4, \ -2 \le y \le 2\}$. The cross-sections of **S** perpendicular to the x-axis are triangles with height $h(x) = x(4-x)$. Find the volume of **S**

A) $\frac{5}{3}$

B) $\frac{10}{3}$

C) $\frac{16}{3}$

D) $\frac{64}{3}$

E) $\frac{28}{3}$

$$\int_0^4 A(x)\,dx$$

$$A(x) = \frac{1}{2}\, b(x) \cdot h(x)$$

$$b(x) = 2 - -2 = 4$$

$$h(x) = x(4-x)$$

$$\int_0^4 2x(4-x)\,dx$$

8) Given that $\int_1^2 xe^x\,dx = e^2$, the value of the integral $\int_1^2 x^2 e^x\,dx$

A) can not be determined
B) is equal to $6e^2 - e$
C) is equal to $2e^2 - e$
D) is equal to $e^2 - 3$
E) is equal to $4e - e^2$

$$\int_1^2 x^2 e^x\,dx$$

$u = x^2$ $\quad e^x\,dx = dv$

$du = 2x\,dx$ $\quad e^x = v$

$$\int_1^2 x^2 e^x\,dx = x^2 e^x \Big|_1^2 - 2\int_1^2 x e^x\,dx$$

$$= 4e^2 - e - 2e^2$$

9) Evaluate:

$$\int_0^{\frac{\pi}{3}} \sec^4 x \, dx = \int_0^{\pi/3} \sec^2 x \sec^2 x \, dx$$

$$= \int_0^{\pi/3} (1 + \tan^2 x) \sec^2 x \, dx$$

let $u = \tan x$, $du = \sec^2 x \, dx$

$$\int_0^{\sqrt{3}} (1+u^2) \, du$$

A) $\frac{4}{3}$

B) $\frac{10}{3}$

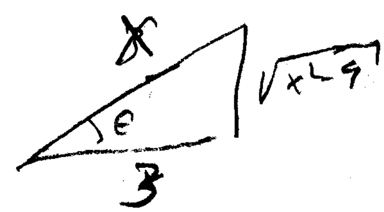
C) $\frac{4}{5}$

D) $\frac{31}{5}$

E) $2\sqrt{3}$

10) Use an appropriate trigonometric substitution to evaluate:

$$\int_4^5 \frac{9}{x^2\sqrt{x^2-9}} \, dx$$



A) $4 - \sqrt{7}$

B) $\frac{4}{5} - \frac{\sqrt{7}}{4}$

C) $\frac{4}{3} - \frac{\sqrt{7}}{3}$

D) $\frac{1}{4} - \frac{1}{\sqrt{7}}$

E) 1

let $x = 3\sec\theta$ $\quad dx = 3\sec\theta\tan\theta \, d\theta$

$$\int \frac{9 \cdot 3 \sec\theta \tan\theta}{3 \cdot 9 \sec^2\theta \tan\theta} \, d\theta$$

$$= \int \cos\theta \, d\theta = \sin\theta + C$$

$$= \frac{\sqrt{x^2-9}}{x} + C$$

$$\left. \frac{\sqrt{x^2-9}}{x} \right|_4^5$$

11) Evaluate

$$\int_3^4 \frac{2}{x(x-1)}\, dx$$

A) $2\ln\frac{3}{4}$

B) $2\ln\frac{1}{2}$

C) $\ln\frac{5}{4}$

D) $\ln\frac{3}{4}$

E) $2\ln\frac{9}{8}$

$$\frac{2}{x(x-1)} = \frac{a}{x} + \frac{b}{x-1}$$

$$= \frac{a(x-1)+bx}{x(x-1)}$$

$$2 = a(x-1) + bx$$

$$a+b=0$$

$$-2=a$$

$$b=2$$

$$\int \left(-\frac{2}{x} + \frac{2}{x-1}\right) dx = 2\ln\left(\frac{x-1}{x}\right)$$

$$2\ln\left(\frac{x-1}{x}\right)\Big|_3^4 = 2\left(\ln\frac{3}{4} - \ln\frac{2}{3}\right)$$

$$= 2\ln\frac{9}{8}$$