1. Which of the following integrals arises when one makes a suitable trigonometric substitution to compute

$$\int \frac{x^2}{\sqrt{4-x^2}}\,dx.$$

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta} = 2\cos\theta$

$= \int \frac{4\sin^2\theta}{2\cos\theta} 2\cos\theta\, d\theta$

$= \int 4\sin^2\theta\, d\theta$

A. $\int 4\sin^2\theta\, d\theta$

B. $\int \frac{2\sin^2\theta}{\cos\theta}\, d\theta$

C. $\int \frac{\tan^2\theta\sec\theta}{4}\, d\theta$

D. $\int \frac{\tan^2\theta}{4\sec\theta}\, d\theta$

E. $\int \frac{\sin^2\theta}{4\cos^2\theta}\, d\theta$

2. Compute $\displaystyle\int_2^4 \frac{dx}{\sqrt{x^2-4}}$.

$x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\, d\theta$

$\sqrt{x^2-4} = 2\tan\theta$

$\int \frac{dx}{\sqrt{x^2-4}} = \int \frac{2\sec\theta\tan\theta\, d\theta}{2\tan\theta}$

$= \int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$

$= \ln\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right| + C$

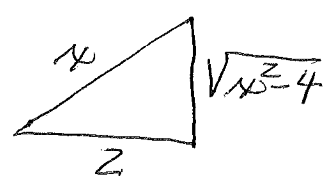
A. $\ln(\sqrt{2}+2)$

B. $\frac{1}{2}\ln(2\sqrt{2}+3)$

C. $\sqrt{2}+\frac{1}{2}\ln(\sqrt{2}+1)$

D. $\ln(2+\sqrt{3})$

E. $2\ln(2\sqrt{2}+3)$



$\int_2^4 \frac{dx}{\sqrt{x^2-4}} = \ln\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right| \Big]_2^4 = \ln\left(2 + \frac{\sqrt{12}}{2}\right)$

$= \ln(2+\sqrt{3})$

3. Evaluate $\displaystyle\int \frac{2x+5}{x^2+2x+2}\,dx$.

$$= \int \frac{2x+2}{x^2+2x+2}\,dx + \int \frac{3}{x^2+2x+2}\,dx$$

$$= \ln|x^2+2x+2| + 3\int \frac{dx}{(x+1)^2+1}$$

$$= \ln|x^2+2x+2) + 3\tan^{-1}(x+1) + C$$

A. $3\ln|x^2+2x+2| + \tan^{-1}(x^2+2x+2) + C$

B. $\ln|x^2+2x+2| + 3\tan^{-1}(x+1) + C$

C. $\ln|x^2+2x+2| + \dfrac{3}{x^2+2x+2} + C$

D. $2x+2+3\tan^{-1}(x+1) + C$

E. $2\ln|2x+2| + 3\tan^{-1}(x+1) + C$

4. What is the form of the partial fraction decomposition of

$$\frac{x+2}{(x-1)^2(x+1)(x^2+4)^2}.$$

A. $\dfrac{A}{(x-1)^2} + \dfrac{B}{x+1} + \dfrac{Cx+D}{(x^2+4)^2}$

B. $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x+1} + \dfrac{D}{x^2+4} + \dfrac{E}{(x^2+4)^2}$

C. $\dfrac{A}{x-1} + \dfrac{B}{x+1} + \dfrac{Cx+D}{x^2+4}$

D. $\dfrac{A}{x-1} + \dfrac{Bx+C}{x^2-1} + \dfrac{Dx+E}{x^2+4} + \dfrac{Fx+G}{(x^2+4)^2}$

E. $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x+1} + \dfrac{Dx+E}{x^2+4} + \dfrac{Fx+G}{(x^2+4)^2}$

5. Evaluate $\displaystyle\int_0^1 \frac{4}{x^2+4x+3}\,dx$.

A. $\ln 2 + \ln 4 - \ln 3$

B. $\frac{1}{2}\ln 2 - \ln 4 + \frac{3}{2}\ln 3$

C. $\frac{1}{2}(\ln 2 - \ln 4 + 3\ln 3)$

D. $2(\ln 2 + 2\ln 4 - \ln 3)$

E. $2(\ln 2 - \ln 4 + \ln 3)$

$$\frac{4}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$$

$$4 = A(x+3) + B(x+1)$$

$$A+B=0 \qquad A=2,\ B=-2$$
$$3A+B=4$$

$$\int_0^1 \frac{4}{x^2+4x+3}\,dx = \int_0^1 \left(\frac{2}{x+1} - \frac{2}{x+3}\right)dx$$

$$= 2\ln|x+1| - 2\ln|x+3|\Big]_0^1$$

$$= 2\left(\ln 2 - \ln 4 + \ln 3\right)$$

6. Evaluate $\int_0^1 \frac{dx}{\sqrt{x}+1}$.

$u = \sqrt{x} \qquad x = u^2$

$dx = 2u\,du$

$\int \frac{dx}{\sqrt{x}+1} = \int \frac{2u\,du}{u+1} = 2\int \frac{u}{u+1}\,du$

$\frac{u}{u+1} = 1 - \frac{1}{u+1}$

$= 2\int\left(1 - \frac{1}{u+1}\right)du = 2\left(u - \ln|u+1|\right) + C$

$= 2\left(\sqrt{x} - \ln(\sqrt{x}+1)\right) + C.$ $\int_0^1 \frac{dx}{\sqrt{x}+1} = 2\left(\sqrt{x} - \ln(\sqrt{x}+1)\right)\Big]_0^1$

$= 2 - 2\ln 2$

A. $1 + \ln 2$

B. $2 - 4\ln 2$

C. $\frac{1}{2} + 2\ln 2$

D. $2 - 2\ln 2$

E. $2 + \frac{1}{2}\ln 2$

7. Find the length of the curve $f(x) = \ln(\sec x)$, $0 \le x \le \pi/3$.

$f'(x) = \frac{1}{\sec x}\sec x \tan x = \tan x$

$ds = \sqrt{1 + (f'(x))^2}\,dx = \sqrt{1+\tan^2 x}\,dx$

$ds = \sec x\,dx$

$s = \int_0^{\pi/3} \sec x\,dx = \ln|\sec x + \tan x|\Big]_0^{\pi/3}$

$= \ln(2 + \sqrt{3})$

A. $\ln(1+\sqrt{2})$

B. $\sqrt{2}+\sqrt{3}$

C. $\ln(\sqrt{2}+\sqrt{3})$

D. $1+\sqrt{2}$

E. $\ln(2+\sqrt{3})$

8. A surface is generated by rotating the curve $y = 2\sqrt{1+x}$, $0 \le x \le 2$, about the x–axis. Find the surface area of the surface.

$$S = \int_0^2 2\pi y \, ds \qquad \frac{dy}{dx} = \frac{1}{\sqrt{1+x}}$$

$$ds = \sqrt{1+(dy/dx)^2} = \sqrt{1+\frac{1}{1+x}}\,dx = \sqrt{\frac{2+x}{1+x}}\,dx$$

$$S = \int_0^2 2\pi\, 2\sqrt{1+x}\,\frac{\sqrt{2+x}}{\sqrt{1+x}}\,dx$$

$$= 4\pi \int_0^2 \sqrt{2+x}\,dx = \frac{8\pi}{3}\Big[(2+x)^{3/2}\Big]_0^2 = \frac{8\pi}{3}(8-2^{3/2})$$

A. $\frac{8\pi}{3}(2-\sqrt{2})$

B. $\frac{4\pi}{3}(\sqrt{3}-\sqrt{2})$

C. $\frac{8\pi}{3}(8-2^{3/2})$

D. $\frac{\pi}{8}(\sqrt{3}-1)$

E. $\frac{8\pi}{3}(8-\sqrt{2})$

9. A lamina of uniform density has the shape of the region bounded by

$$y = x, \quad \text{and} \quad y = x^4.$$

The area of the region is $\frac{3}{10}$. Which expression gives the y–coordinate $\bar{y}$ of the center of mass.

$$M_x = \int_0^1 \frac{(x)^2 - (x^4)^2}{2}\,dx$$

$$M_x = \frac{1}{2}\int_0^1 (x^2 - x^8)\,dx$$

$$\bar{y} = \frac{M_x}{3/10} = \frac{10}{3}\,\frac{1}{2}\int_0^1 (x^2-x^8)\,dx$$

$$= \frac{5}{3}\int_0^1 (x^2-x^8)\,dx$$

A. $\bar{y} = \frac{10}{3}\int_0^1 (x^2 - x^5)\,dx$

B. $\bar{y} = \frac{5}{3}\int_0^1 (x - x^4)^2\,dx$

C. $\bar{y} = \frac{10\pi}{3}\int_0^1 (x^2 - x^5)\,dx$

D. $\bar{y} = \frac{5}{3}\int_0^1 (x^2 - x^8)\,dx$

E. $\bar{y} = \frac{10\pi}{3}\int_0^1 (x^2 - x^8)\,dx$

10. The curve $y = e^x$, $0 \leq x \leq 2$, is rotated about the y–axis. Which integral gives the surface area of the surface of revolution.

$$S = \int_0^2 2\pi x \, ds$$

$$\frac{dy}{dx} = e^x$$

$$ds = \sqrt{1 + (dy/dx)^2}\, dx = \sqrt{1+e^{2x}}\, dx$$

$$S = \int_0^2 2\pi x \sqrt{1+e^{2x}}\, dx$$

A. $\int_0^2 2\pi e^x \sqrt{1+e^{2x}}\, dx$

B. $\int_0^2 2\pi x \sqrt{1+e^{2x}}\, dx$

C. $\int_0^2 2\pi x e^x \, dx$

D. $\int_0^2 \pi x e^{2x} \sqrt{1+e^{2x}}\, dx$

E. $\int_0^2 \pi e^{2x}\, dx$

11. Which statement is true about the following improper integrals.

I. $\int_{-1}^{1} \frac{1}{x}\, dx$ II. $\int_1^{\infty} \frac{1}{e^x}\, dx$ III. $\int_{\pi}^{\infty} \frac{\sin^2 x}{x^2}\, dx$

I. $\int_{-1}^{1} \frac{1}{x}\, dx = \int_{-1}^{0} \frac{1}{x}\, dx + \int_0^1 \frac{1}{x}\, dx$

$\int_{-1}^{0} \frac{1}{x}\, dx = \lim_{c \to 0^-} \ln|c| = -\infty.$

II. $\int_1^{\infty} \frac{1}{e^x}\, dx = \lim_{b\to\infty}\left(\frac{-1}{e^b} + \frac{1}{e}\right) = \frac{1}{e}.$

III. $\frac{\sin^2 x}{x^2} \leq \frac{1}{x^2}$ AND $\int_{\pi}^{\infty} \frac{1}{x^2}\, dx = \lim_{b\to\infty}\left(\frac{-1}{b} + \frac{1}{\pi}\right) = \frac{1}{\pi}$

BY THE COMPARISON TEST, $\int_{\pi}^{\infty} \frac{\sin^2 x}{x^2}\, dx$ CONVERGES.

A. II and III converge. I diverges.

B. I and II converge. III diverges.

C. II converges. I and III diverge.

D. I, II and III converge.

E. I, II and III diverge.