MA162 — EXAM II — FALL 2016 — OCTOBER 20, 2016
TEST NUMBER 01

INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. **MARK YOUR TEST NUMBER ON YOUR SCANTRON**
4. Once you are allowed to open the exam, make sure you have a complete test. There are 7 different test pages (including this cover page).
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. **Circle your answers on this test booklet.**
6. Each problem is worth 100/12 points. The maximum possible score is 100 points. No partial credit.
7. Do not leave the exam room during the first 20 minutes of the exam.
8. If you do not finish your exam in the first 50 minutes, you must wait until the end of the exam period to leave the room.
9. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

DON'T BE A CHEATER:

1. Do not give, seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.
2. Do not look at the exam or scantron of another student.
3. Do not allow other students to look at your exam or your scantron.
4. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
5. Do not consult notes or books.
6. **Do not handle** phones or cameras, calculators or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: SOLUTIONS

STUDENT SIGNATURE: ______________________

STUDENT ID NUMBER: ______________________

SECTION NUMBER AND RECITATION INSTRUCTOR: ______________________

1. Compute $\int_0^3 \frac{dx}{(x^2+9)^{\frac{3}{2}}}$.

A. $\frac{\sqrt{2}}{6}$

B. $\frac{\sqrt{2}}{9}$

C. $\sqrt{2}$

D. $\frac{\sqrt{2}}{27}$

E. $\boxed{\frac{\sqrt{2}}{18}}$

$x = 3\tan\theta \quad , \quad dx = 3\sec^2\theta\, d\theta$

$$= \int_0^{\pi/4} \frac{3\sec^2\theta\, d\theta}{(9\sec^2\theta)^{3/2}} =$$

$$= \int_0^{\pi/4} \frac{3\sec^2\theta\, d\theta}{27\sec^3\theta\, d\theta} = \frac{1}{9}\int_0^{\pi/4} \cos\theta\, d\theta$$

$$= \frac{1}{9}\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{18}$$

2. Compute $\int_{\frac{1}{\sqrt{2}}}^{1} \frac{dx}{\sqrt{4x^2-1}}$.

A. $\frac{1}{2}\ln\left(\frac{1+\sqrt{2}}{1+\sqrt{3}}\right)$

B. $\frac{1}{3}\ln\left(\frac{1+\sqrt{3}}{1+\sqrt{2}}\right)$

C. $\boxed{\frac{1}{2}\ln\left(\frac{2+\sqrt{3}}{1+\sqrt{2}}\right)}$

D. $\frac{1}{3}\ln\left(\frac{1+\sqrt{3}}{2+\sqrt{2}}\right)$

E. $\frac{1}{4}\ln\left(\frac{2+\sqrt{3}}{1+\sqrt{2}}\right)$

$x = \frac{1}{2}\sec\theta \quad , \quad dx = \frac{1}{2}\sec\theta\tan\theta\, d\theta$

$$= \int_{\theta_0}^{\theta_1} \frac{\frac{1}{2}\sec\theta\tan\theta}{\tan\theta}\, d\theta$$

$\sec\theta_0 = \sqrt{2}$

$\sec\theta_1 = 2$

$$= \frac{1}{2}\int_{\theta_0}^{\theta_1} \sec\theta\, d\theta$$

$$= \frac{1}{2}\ln|\sec\theta + \tan\theta|\Big|_{\theta_0}^{\theta_1}$$

$$= \frac{1}{2}\ln|2+\sqrt{3}| - \frac{1}{2}\ln|\sqrt{2}+1|$$

$$= \frac{1}{2}\ln\frac{2+\sqrt{3}}{1+\sqrt{2}}$$

3. Compute $\int_1^2 \frac{dx}{x(x^2+1)}$

A. $\frac{1}{3}\ln(\frac{1}{3})$

$\boxed{\text{B. } \frac{1}{2}\ln(\frac{8}{5})}$

C. $\frac{3}{4}\ln(\frac{2}{3})$

D. $\frac{1}{4}\ln(\frac{5}{8})$

E. $\frac{2}{3}\ln(\frac{6}{5})$

$$\frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{x^2+1}$$

$$\int_1^2 \frac{dx}{x(x^2+1)} = \int_1^2 \frac{dx}{x} - \frac{1}{2}\int_1^2 \frac{2x\,dx}{x^2+1}$$

$$= \ln 2 - \frac{1}{2}\int_2^5 \frac{du}{u}$$

$$= \ln 2 - \frac{1}{2}(\ln 5 - \ln 2)$$

$$= \frac{3}{2}\ln 2 - \frac{1}{2}\ln 5 = \frac{1}{2}\ln(8/5)$$

4. Compute $\int_2^3 \frac{dx}{x^2+x-2}$

A. $\ln 2 + \ln 3$

B. $\frac{2}{3}\ln 2 - \frac{1}{3}\ln 5$

C. $2\ln 2 - \frac{1}{3}\ln 5$

D. $\frac{1}{3}\ln 2 - \frac{1}{3}\ln 5$

$\boxed{\text{E. } \ln 2 - \frac{1}{3}\ln 5}$

$$\frac{1}{x^2+x-2} = \frac{1}{(x+2)(x-1)} = \frac{1}{3}\frac{1}{x-1} - \frac{1}{3}\frac{1}{x+2}$$

$$\int_2^3 \frac{dx}{x^2+x-2} = \frac{1}{3}\int_2^3 \frac{dx}{x-1} - \frac{1}{3}\int_2^3 \frac{dx}{x+2}$$

$$= \frac{1}{3}\ln 2 - \frac{1}{3}(\ln 5 - \ln 4)$$

$$= \frac{1}{3}\ln 2 - \frac{1}{3}\ln 5 + \frac{2}{3}\ln 2$$

$$= \ln 2 - \frac{1}{3}\ln 5$$

5. Compute the indefinite integral $\int_e^{\infty} \frac{1}{x(\ln x)^3} dx$

$u = \ln x$

$\boxed{\text{A. } \frac{1}{2}}$

B. 2

C. $\frac{3}{4}$

D. $\frac{2}{3}$

E. $\frac{1}{5}$

$$= \int_1^{\infty} u^{-3}\, du = -\frac{1}{2} u^{-2} \Big|_1^{\infty} = 1/2.$$

6. Which of the following indefinite integrals converge?

✓ NO ✓

$$I. \int_0^1 \frac{dx}{\sqrt{x}}, \quad II. \int_0^2 \frac{dx}{x-1} \quad III. \int_{10}^{\infty} \frac{dx}{x^2+1}$$

A. I, II and III

B. only I

C. only II

D. only II and III

$\boxed{\text{E. only I and III}}$

$$\int_0^1 x^{-1/2}\, dx = 2x^{1/2}\Big|_0^1 = 2.$$

$$\int_0^1 \frac{dx}{x-1} = -\int_{-1}^{0} \frac{du}{u} \quad \text{diverges}$$

$u = x-1$

$$\int_{10}^{N} \frac{dx}{x^2+1} = \arctan x \Big|_{10}^{N}$$

$$= \arctan N - \arctan 10$$

$$\lim_{N\to\infty} \int_{10}^{\infty} \frac{dx}{x^2+1} = \pi/2 - \arctan 10$$

7. Find the length of curve $y = f(x)$ from $x = 0$ to $x = \pi/3$ if $f'(x) = \sqrt{\sec^4 x - 1}$.

A. 2

B. $\frac{\sqrt{3}}{3}$

C. 4

D. $\boxed{\sqrt{3}}$

E. $\sqrt{2}$

$$L = \int_0^{\pi/3} \sqrt{\sec^4 x}\, dx = \int_0^{\pi/3} \sec^2 x\, dx$$

$$= \tan x \Big|_0^{\pi/3} = \frac{\sin \pi/3}{\cos \pi/3} = \frac{\sqrt{3}/2}{1/2}$$

$$= \sqrt{3}$$

8. The area of the surface obtained by revolving the curve $y = 4\sqrt{x}$ from $(0,0)$ to $(1,4)$ about the x-axis is equal to:

A. $\frac{16\pi}{3}(5\sqrt{5} - 4)$

B. $\boxed{\frac{16\pi}{3}(5\sqrt{5} - 8)}$

C. $8\pi(5\sqrt{5} - 2)$

D. $\frac{8\pi}{3}(5\sqrt{5} - 4)$

E. $\frac{4\pi}{3}(5\sqrt{5} - 1)$

$$A = 2\pi \int_0^1 4\sqrt{x} \sqrt{1 + \left(\frac{2}{\sqrt{x}}\right)^2} =$$

$$= 8\pi \int_0^1 \sqrt{x+4}\, dx \qquad x + 4 = u$$

$$= 8\pi \int_4^5 u^{1/2} du = 8\pi \cdot \frac{2}{3} u^{3/2} \Big|_4^5$$

$$= \frac{16\pi}{3}(5\sqrt{5} - 8)$$

9. Find the centroid of region bounded by $y = x$ and $y = x^2$.

$$A = \int_0^1 (x - x^2)\,dx = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$M_y = \int_0^1 (x^2 - x^3)\,dx = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

$$M_x = \frac{1}{2}\int_0^1 x^2 - x^4 = \frac{1}{2} \cdot \left(\frac{1}{3} - \frac{1}{5}\right) = \frac{1}{15}$$

A. $(1/2,\ 2/5)$

B. $(2/3,\ 2)$

C. $(1/2,\ 5/3)$

D. $(2,\ 3/2)$

E. $(3/2,\ 5/3)$

$$\bar{x} = \frac{1}{2}\ ;\ \bar{y} = \frac{6}{15} = \frac{2}{5}.$$

10. Evaluate $\lim_{n\to\infty} \left(\frac{2n^3 + 3n^2 - 2}{n^2} - 2n\right)$: $= \lim_{n\to\infty} \frac{\cancel{2n^3} + 3n^2 - 2 - \cancel{2n^3}}{n^2}$

$$= \lim_{n\to\infty} \frac{3n^2 - 2}{n^2}$$

A. 2

B. 0

C. ∞

$$= \lim_{n\to\infty} 3 - 2/n^2 = 3$$

D. the limit does not exist

E. 3

11. $\sum_{k=0}^{\infty} \frac{2+3^k}{4^k} =$

$$2 \sum_{k=0}^{\infty} \frac{1}{4^k} + \sum_{k=0}^{\infty} \left(\frac{3}{4}\right)^k =$$

$$= 2 \frac{1}{1-1/4} + \frac{1}{1-3/4}$$

$$= \frac{2}{3/4} + \frac{1}{1/4} = 4 + 8/3 = \frac{20}{3}$$

A. ∞

B. $\frac{20}{3}$

C. 4

D. $\frac{8}{3}$

E. $\frac{14}{3}$

12. $\sum_{n=1}^{\infty} \frac{2}{n(n+2)} =$

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right) = 1 + \frac{1}{2} = 3/2.$$

A. $\frac{4}{3}$

B. ∞

C. $\boxed{\frac{3}{2}}$

D. 1

E. 3

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SECTION NUMBER AND RECITATION INSTRUCTOR: ______________________

1. Compute $\displaystyle\int_0^2 \frac{dx}{(x^2+4)^{\frac{3}{2}}}$.

A. $\frac{\sqrt{2}}{8}$

B. $\frac{\sqrt{2}}{4}$

C. $\frac{\sqrt{2}}{2}$

D. $\frac{\sqrt{2}}{32}$

E. $\frac{\sqrt{2}}{16}$

$x = 2\tan\theta \qquad dx = 2\sec^2\theta\, d\theta$

$$= \int_0^{\pi/4} \frac{2\sec^2\theta}{8\sec^3\theta}\, d\theta = \frac{1}{4}\int_0^{\pi/4} \cos\theta\, d\theta$$

$$= \frac{\sqrt{2}}{8}$$

2. Compute $\displaystyle\int_{\frac{2}{\sqrt{3}}}^2 \frac{dx}{\sqrt{x^2-1}}$.

A. $\ln\left(\frac{1+\sqrt{3}}{1+\sqrt{2}}\right)$

B. $\ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$

C. $\ln\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)$

D. $\ln\left(\frac{2+\sqrt{3}}{1+\sqrt{3}}\right)$

E. $\ln\left(\frac{2+\sqrt{2}}{2+\sqrt{3}}\right)$

$\tan\theta_1 = \sqrt{\sec^2\theta_1 - 1} = \sqrt{3}$

$\tan\theta_0 = \sqrt{\sec^2\theta_0 - 1} = 1/\sqrt{3}$

$\sec\theta_0 = 2/\sqrt{3}, \quad \sec\theta_1 = 2$

$x = \sec\theta, \quad dx = \sec\theta\tan\theta\, d\theta$

$$= \int_{\theta_0}^{\theta_1} \frac{\sec\theta\tan\theta}{\tan\theta}\, d\theta = \int_{\theta_0}^{\theta_1} \sec\theta\, d\theta$$

$$= \ln|\sec\theta + \tan\theta|\Big|_{\theta_0}^{\theta_1}$$

$$= \ln|2+\sqrt{3}| - \ln\left|\frac{2}{\sqrt{3}} + \sqrt{\frac{4}{3}-1}\right|$$

$$= \ln|2+\sqrt{3}| - \ln\sqrt{3}$$

$$= \ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$$

3. Compute $\displaystyle\int_1^2 \frac{dx}{x(x^2+2)}$ $= \displaystyle\frac{1}{2}\int_1^2 \frac{1}{x} - \frac{1}{4}\int_1^2 \frac{2x}{x^2+2} = \frac{1}{2}\ln 2 - \frac{1}{4}(\ln 6 - \ln 3)$

$= \frac{1}{2}\ln 2 - \frac{1}{4}\ln 2 = \frac{1}{4}\ln 2.$

A. $\frac{1}{3}\ln 2$

B. $\frac{1}{2}\ln 2$

C. $\frac{3}{4}\ln 2$

D. $\frac{1}{4}\ln 2$

E. $\frac{2}{3}\ln 2$

$$\frac{1}{x^2+x-6} = \frac{1}{(x+3)(x-2)} =$$

$$= \frac{1}{5}\frac{1}{x-2} - \frac{1}{5}\frac{1}{x+3}$$

4. Compute $\displaystyle\int_3^4 \frac{dx}{x^2+x-6}$

$$= \frac{1}{5}\int_3^4 \frac{dx}{x-2} - \frac{1}{5}\int_3^4 \frac{dx}{x+3}$$

$$= \frac{1}{5}\ln 2 - \frac{1}{5}(\ln(7) - \ln 6)$$

$$= \frac{1}{5}\ln 2 - \frac{1}{5}\ln 7/6$$

A. $\frac{1}{5}\ln 2 - \frac{1}{5}\ln(\frac{5}{2})$

B. $\frac{1}{5}\ln 2 - \frac{1}{5}\ln(\frac{7}{6})$

C. $\frac{3}{4}\ln 2 + \frac{3}{4}\ln 3$

D. $\frac{1}{2}\ln 2 - \frac{1}{2}\ln 7$

E. $\frac{1}{4}\ln 2 - \frac{1}{4}\ln(\frac{7}{3})$

5. Compute the indefinite integral $\int_e^\infty \frac{1}{x(\ln x)^5}\,dx$

$u = \ln x$

A. $\frac{1}{2}$

B. 2

C. $\frac{1}{4}$

D. $\frac{2}{3}$

E. $\frac{1}{5}$

$$= \int_1^\infty u^{-5}\,du = -\frac{1}{4}u^{-4}\Big|_1^\infty = 1/4$$

6. Which of the following indefinite integrals converge?

$$\overset{F}{I.\ \int_0^1 \frac{dx}{x^2-1}},\quad \overset{F}{II.\ \int_0^2 \frac{1}{x}\,dx}\quad \overset{T}{III.\ \int_{10}^\infty \frac{dx}{x^2}}$$

A. I, II and III

B. none

C. only III

D. only I and III

E. only II

$$\int_0^1 \frac{dx}{x^2-1} = \int_0^1 \frac{dx}{(x+1)(x-1)} = -\int_0^1 \frac{dx}{(x+1)(1-x)}$$

$$\int_0^1 \frac{dx}{(x+1)(1-x)} \geq \frac{1}{2}\int_0^1 \frac{dx}{1-x} \quad \text{diverges}$$

$$\int_0^1 \frac{1}{x^p}\,dx \quad \text{diverges if } p \geq 1$$

7. Find the length of curve $y = f(x)$ from $x = 0$ to $x = \pi/3$ if $f'(x) = \sqrt{\sec^2 x \tan^2 x - 1}$.

A. 2

B. $\sqrt{3}/3$

C. 1

D. 4

E. $\sqrt{2}$

$$\int_0^{\pi/3} \sqrt{\sec^2 x \tan^2 x}\, dx = \int_0^{\pi/3} \sec x \tan x \, dx$$

$$= \sec x \Big|_0^{\pi/3} = \frac{1}{1/2} - 1 = 1$$

8. The area of the surface obtained by revolving the curve $y = 2\sqrt{x}$ from $(0,0)$ to $(1,2)$ about the $x-$axis is equal to

A. $\frac{2\pi}{3}(2\sqrt{2} - 1)$

No B. $\frac{8\pi}{3}(\sqrt{2} - 1)$

C. $8\pi(2\sqrt{2} - 1)$

D. $\frac{8\pi}{3}(2\sqrt{2} - 1)$ CORRECT.

E. $\frac{4\pi}{3}(\sqrt{2} - 1)$

$$A = 2\pi \int_0^1 2\sqrt{x} \sqrt{1 + \frac{1}{x}}\, dx$$

$$= 4\pi \int_0^1 \sqrt{1+x}\, dx \qquad 1 + x = u$$

$$= 4\pi \int_1^2 u^{1/2}\, du = 4\pi \cdot \frac{2}{3} u^{3/2} \Big|_1^2$$

$$= \frac{8\pi}{3}(2\sqrt{2} - 1)$$

9. Find the centroid of region bounded by $y = x$ and $y = x^3$, and $x \geq 0$.

A. (8/15, 4/21)

B. (4/15, 4/21)

C. (2/15, 2/21)

D. (1/15, 1/21)

E. (8/15, 8/21)

$A = \int_0^1 (x - x^3)\,dx = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$M_y = \int_0^1 x^2 - x^4\,dx = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$

$M_x = \frac{1}{2}\int_0^1 x^2 - x^6 = \frac{1}{2}\left(\frac{1}{3} - \frac{1}{7}\right)$

$= \frac{1}{2} \cdot \frac{4}{21} = \frac{2}{21}.$

$\bar{x} = \frac{2}{15} \times 4 = \frac{8}{15}$

$\bar{y} = \frac{2}{21} \times 4 = \frac{8}{21}$

10. Evaluate $\lim_{n\to\infty} \left(\frac{n^3 + 2n^2 - 3}{n^2} - n\right)$

$= \lim_{n\to\infty} \left(\frac{n^3 + 2n^2 - 3 - n^3}{n^2}\right)$

$= \lim_{n\to\infty} \frac{2n^2 - 3}{n^2} =$

$= \lim_{n\to\infty} 2 - 3/n^2 = 2.$

A. 2

B. 0

C. ∞

D. the limit does not exist

E. 1

11. Evaluate $\sum_{k=0}^{\infty} \frac{2+2^k}{3^k}$ $= 2 \sum_{k=0}^{\infty} \frac{1}{3^k} + \sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k$

$= 2 \frac{1}{1-1/3} + \frac{1}{1-2/3} = \frac{2}{2/3} + \frac{1}{1/3}$

$= 3 + 3 = 6$

A. ∞

B. 3

C. 6

D. $\frac{8}{3}$

E. $\frac{14}{3}$

12. $\sum_{n=1}^{\infty} \frac{1}{n(n+2)} =$ $\frac{1}{2} \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right) = \frac{1}{2}\left(1 + \frac{1}{2}\right) = 3/4$

A. $\frac{3}{4}$

B. ∞

C. $\frac{3}{2}$

D. 1

E. 3