Exam 2, Math 162, Spring 2014, (version 01)

#1. $$\frac{x+1}{x^3(x^2+x+3)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+x+3} + \frac{Fx+G}{(x^2+x+3)^2}.$$

E

#2.
$$\begin{array}{r|l} & 2 \\ x^2+2x+2 & 2x^2 \\ & -(2x^2+4x+4) \\ \hline & -(4x+4) \end{array}$$

$$\int \frac{2x^2}{x^2+2x+2}\,dx = 2\int dx - \int \frac{4x+4}{x^2+2x+2}\,dx, \quad \boxed{\begin{array}{l} u = x^2+2x+2 \\ du = (2x+2)\,dx \end{array}}$$

$$= 2x - 2\int \frac{du}{u} = 2x - 2\ln|x^2+2x+2| + C.$$

C

#3.
$$\int_0^2 (\ln x + x)\,dx = \lim_{a\to 0^+} \int_a^2 (\ln(x) + x)\,dx = \lim_{a\to 0^+} \left[x\ln x - x + \frac{x^2}{2} \Big|_a^2 \right]$$

$$= \lim_{a\to 0^+} \left[2\ln(2) - a\ln(a) + a - \frac{a^2}{2} \right] = 2\ln(2).$$

E

#4.

$$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$$

$$\text{Length} = \int_0^1 \sqrt{1 + \frac{9}{4}x}\, dx, \qquad u = 1 + \frac{9}{4}x, \quad du = \frac{9}{4}dx$$

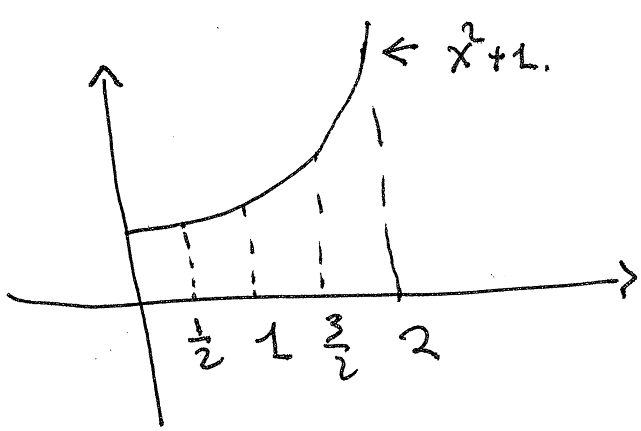
$$= \frac{4}{9}\int_1^{\frac{13}{4}} \sqrt{u}\, du = \frac{4}{9}\cdot\frac{2}{3} u^{3/2}\Big|_1^{\frac{13}{4}} = \frac{4}{9}\cdot\frac{2}{3}\left[\left(\frac{13}{4}\right)^{3/2} - 1\right]$$

$$= \frac{8}{27}\left[\frac{13^{3/2}}{8} - 1\right] = \frac{1}{27}\left[13^{3/2} - 8\right] \qquad \text{B}$$

#5) $$\int_0^2 (x^2+1)\,dx \approx T_4 = \frac{1}{4}\left[0^2+1 + 2\left[\left(\tfrac{1}{2}\right)^2+1\right] + 2\left[1^2+1\right] + 2\left[\left(\tfrac{3}{2}\right)^2+1\right] + \left[2^2+1\right]\right]$$

$$= \frac{1}{4}\left[1 + 2\left[\frac{1}{4}+1\right] + 4 + 2\left[\frac{9}{4}+1\right] + 5\right] = \frac{1}{4}\left[14 + \frac{1}{2} + \frac{9}{2}\right]$$

$$= \frac{1}{4}[19] = \frac{19}{4}$$



D

#6.

$$\frac{dy}{dx} = \frac{1}{2}(1+4x)^{-1/2} \cdot 4 = \frac{2}{\sqrt{1+4x}}$$

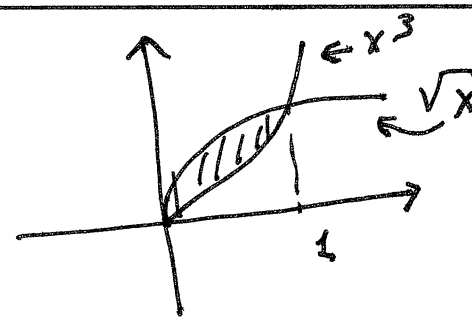
$$\text{Surface area} = \int_1^5 2\pi\sqrt{1+4x}\,\sqrt{1+\frac{4}{1+4x}}\,dx$$

$$= 2\pi\int_1^5 \sqrt{1+4x}\,\sqrt{\frac{1+4x+4}{1+4x}}\,dx = 2\pi\int_1^5 \sqrt{4x+5}\,dx$$

$$= 2\pi\left[\frac{2}{3}\cdot\frac{1}{4}(4x+5)^{3/2}\Big|_1^5\right] = \frac{\pi}{3}\left[25^{3/2} - 9^{3/2}\right]$$

D

#7.



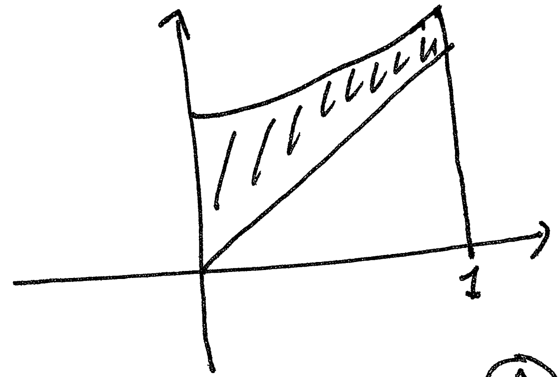
$$\bar{x} = \frac{1}{A}\int_0^1 x(\sqrt{x}-x^3)\,dx$$

$$= \frac{1}{A}\int_0^1 (x^{3/2}-x^4)\,dx$$

$$= \frac{1}{A}\left[\frac{2}{5}x^{5/2} - \frac{x^5}{5}\Big|_0^1\right] = \frac{1}{A}\left[\frac{2}{5}-\frac{1}{5}\right] = \frac{1}{5A}$$

E

#8



$$\bar{Y} = \frac{1}{2A}\int_0^1 \left[(1+x^2) - x^2\right]dx = \frac{1}{2A}.$$

#9. $$\lim_{n\to\infty}\left(\sqrt{\frac{n^2+3n}{4n^2+1}} - \frac{3n+1}{e^n}\right) = \lim_{n\to\infty}\sqrt{\frac{n^2+3n}{4n^2+1}} - \lim_{n\to\infty}\left(\frac{3n+1}{e^n}\right)$$

- $$\lim_{n\to\infty}\left(\frac{3n+1}{e^n}\right) = 0$$

- $$\lim_{n\to\infty}\sqrt{\frac{n^2+3n}{4n^2+1}} = \lim_{n\to\infty}\sqrt{\frac{\cancel{n^2}\left(1+\frac{3}{n}\right)}{\cancel{n^2}\left(4+\frac{1}{n^2}\right)}} = \sqrt{\frac{1}{4}} = \frac{1}{2}.$$

C

#10. $$\sum_{n=1}^{\infty}\frac{3^{n-1}}{4^n} = \frac{1}{4}\sum_{n=1}^{\infty}\frac{3^{n-1}}{4^{n-1}} = \frac{1}{4}\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^{n-1}$$

$$= \frac{1}{4}\left[\frac{1}{1-\frac{3}{4}}\right] = 1$$

E

#11 I. Converses, (p-series, $p = \frac{5}{4} > 1$)

II. Diverges, (Integral test)

III. Diverges, (Integral test)

A

#12. I. Converses (Integral test)

II. Diverses ($\frac{2n}{n+1} \longrightarrow 2$)

III. Converges (Geometric series, $r = \frac{1}{e}$)

D