MA162 — EXAM II — SPRING 2017 — MARCH 9, 2017
TEST NUMBER 01

INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. MARK YOUR TEST NUMBER ON YOUR SCANTRON
4. Once you are allowed to open the exam, make sure you have a complete test. There are 7 different test pages (including this cover page).
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers on this test booklet.
6. There are 14 problems and the number of points each problem is worth is indicated next to the problem number. The maximum possible score is 100 points. No partial credit.
7. Do not leave the exam room during the first 20 minutes of the exam.
8. If you do not finish your exam in the first 50 minutes, you must wait until the end of the exam period to leave the room.
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3. Do not allow other students to look at your exam or your scantron.
4. You may not compare answers with anyone else or consult another student until after you have finished your exam, given it to your instructor and left the room.
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I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: SOLUTIONS

STUDENT SIGNATURE: ____________________

STUDENT ID NUMBER: ____________________

SECTION NUMBER AND RECITATION INSTRUCTOR: ____________________

1. (8 points) Compute $\int_0^1 xe^{2x}\,dx$.

$\boxed{\text{A. } \frac{1+e^2}{4}}$

B. $\frac{1+3e^2}{2}$

C. $1+2e^2$

D. $1+3e^2$

E. $\frac{1+2e^2}{4}$

$u = x,\ dv = e^{2x},\ du = dx$

$v = \frac{1}{2}e^{2x}$

$\int x e^{2x}\,dx = \frac{x}{2}e^{2x} - \frac{1}{2}\int e^{2x}\,dx$

$= \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x}.$

So $\int_0^1 x e^{2x}\,dx = \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x}\Big|_0^1$

$= \left(\frac{e^2}{2} - \frac{1}{4}e^2\right) - \left(0 - \frac{1}{4}\right)$

$= \cancel{3}\frac{1}{4}e^2 + \frac{1}{4} = \frac{1+e^2}{4}$

2. (8 points) Compute $\int_0^{\frac{\pi}{4}} 5(\sec^4 x)(\tan^2 x)\,dx$.

A. $\frac{3}{4}$

B. $\frac{3}{8}$

$\boxed{\text{C. } \frac{8}{3}}$

D. $\frac{8}{5}$

E. $\frac{5}{8}$

$\sec^2 x = 1 + \tan^2 x$

$= 5\int_0^{\pi/4} \sec^2 x\,(1+\tan^2 x)\tan^2 x\,dx$

$u = \tan x$

$du = \sec^2 x$

$= 5\int_0^1 u^2(1+u^2)\,du = 5\left(\frac{u^3}{3} + \frac{u^5}{5}\right)\Big|_0^1$

$= 5\left(\frac{1}{3} + \frac{1}{5}\right) = 5 \cdot \frac{8}{15} = \frac{8}{3}$

3. (8 points) If we use a trigonometric substitution to evaluate $\int \frac{\sqrt{x^2+4}}{x^2}\,dx$ the integral becomes

A. $\int \frac{\sec\theta}{2\tan^2\theta}\,d\theta$

B. $\int \frac{\tan\theta}{4\sec^2\theta}\,d\theta$

C. $\int \frac{\sec^2\theta}{\tan\theta}\,d\theta$

D. $\boxed{\int \frac{\sec^3\theta}{\tan^2\theta}\,d\theta}$

E. $\int \frac{\tan^2\theta}{\sec\theta}\,d\theta$

$x = 2\tan\theta$, $dx = 2\sec^2\theta\,d\theta$

$x^2+4 = 4\tan^2\theta + 4 = 4(1+\tan^2\theta)$

$= 4\sec^2\theta.$

$$\int \frac{\sqrt{x^4+4}}{x^2}\,dx = \int \frac{2\sec\theta \cdot 2\sec^2\theta\,d\theta}{4\tan^2\theta}$$

$$= \int \frac{\sec^3\theta}{\tan^2\theta}\,d\theta.$$

4. (8 points) Compute $\int_0^{\frac{1}{2}} \frac{x^3}{(1-x^2)^{\frac{3}{2}}}\,dx$.

A. $\frac{5\sqrt{3}-8}{6}$

B. $\frac{4\sqrt{3}-5}{6}$

C. $\frac{4\sqrt{3}-3}{6}$

D. $\frac{6\sqrt{3}-7}{6}$

E. $\boxed{\frac{7\sqrt{3}-12}{6}}$

$x = \sin\theta$, $dx = \cos\theta\,d\theta$

$$\int_0^{1/2} \frac{x^3}{(1-x^2)^{3/2}}\,dx = \int_0^{\pi/6} \frac{\sin^3\theta \cdot \cos\theta\,d\theta}{\cos^3\theta} =$$

$$= \int_0^{\pi/6} \frac{\sin^3\theta}{\cos^2\theta}\,d\theta = \int_0^{\pi/6} \sin\theta\left(\frac{1-\cos^2\theta}{\cos^2\theta}\right)d\theta$$

Set $\cos\theta = u$

$du = -\sin\theta\,d\theta$

$$= \int_{\sqrt{3}/2}^{1} \left(\frac{1-u^2}{u^2}\right)du = \int_{\sqrt{3}/2}^{1} (u^{-2}-1)\,du$$

$$= -(u^{-1}+u)\Big|_{\sqrt{3}/2}^{1} = -\left[2 - \left(\frac{2}{\sqrt{3}} + \frac{\sqrt{3}}{2}\right)\right] = \frac{4\sqrt{3}-7}{2\sqrt{3}} = \frac{7\sqrt{3}-12}{6}.$$

5. (8 points) Find the length of the curve $y = \frac{3}{8}\left(x^{4/3} - 2x^{2/3}\right)$ for $0 \le x \le 1$. The arc-length formula holds in this case.

A. $\frac{5}{8}$

B. $\frac{13}{8}$

C. $\frac{7}{8}$

D. $\boxed{\frac{9}{8}}$

E. $\frac{11}{8}$

$$y' = \frac{3}{8}\cdot\frac{4}{3}x^{1/3} - \frac{3}{4}\cdot\frac{2}{3}x^{-1/3}$$

$$y' = \frac{1}{2}x^{1/3} - \frac{1}{2}x^{-1/3}$$

$$1+(y')^2 = 1 + \frac{1}{4}\left(x^{1/3} - x^{-1/3}\right)^2 =$$

$$= 1 + \frac{1}{4}x^{2/3} + \frac{1}{4}x^{-2/3} - \frac{1}{2} = \frac{1}{4}x^{2/3} + \frac{1}{4}x^{-2/3} + \frac{1}{2}$$

$$= \frac{1}{4}\left(x^{1/3} + x^{-1/3}\right)^2.$$

$$\sqrt{1+(y')^2} = \frac{1}{2}\left(x^{1/3} + x^{-1/3}\right).$$

$$L = \int_0^1 \frac{1}{2}\left(x^{1/3} + x^{-1/3}\right)dx = \frac{1}{2}\cdot\frac{3}{4}x^{4/3} + \frac{1}{2}\cdot\frac{3}{2}x^{2/3}\Big|_0^1$$

$$= \frac{3}{8} + \frac{3}{4} = \frac{9}{8}$$

6. (8 points) Which of the following improper integrals are convergent?

I. $\displaystyle\int_0^\infty xe^{-x^2}\,dx$ II. $\displaystyle\int_0^\infty \frac{1}{1+x^2}\,dx$ III. $\displaystyle\int_3^\infty \frac{1}{2-x}\,dx$

$\boxed{\text{A. I and II only}}$

B. II and III only

C. II only

D. None of them

E. All of them

$$\int_0^\infty x e^{-x^2} dx = \int_0^\infty \frac{1}{2} e^{-u} du$$

$$= \frac{1}{2}.$$

$x^2 = u$

$$\int_0^\infty \frac{1}{1+x^2} = \tan^{-1}x \Big|_0^\infty = \pi/2.$$

$$\int_3^\infty \frac{1}{2-x}dx = -\int_1^\infty \frac{du}{u} = \text{diverges}$$

$-2+x=u$

7. (8 points) Compute the integral $\int_1^2 \frac{1}{x^2+4x+3}\,dx$.

$x^2+4x+3 = (x+1)(x+3)$

A. $\frac{1}{2}\ln(\frac{3}{4})$

B. $\frac{1}{2}\ln(\frac{2}{5})$

C. $\boxed{\frac{1}{2}\ln(\frac{6}{5})}$

D. $\frac{1}{2}\ln(\frac{3}{5})$

E. $\frac{1}{2}\ln(\frac{3}{8})$

$$\frac{1}{x^2+4x+3} = \frac{1}{2}\left(\frac{1}{x+1} - \frac{1}{x+3}\right)$$

$$\int_1^2 \frac{1}{x^2+4x+3}\,dx = \frac{1}{2}\left(\ln|x+1| - \ln|x+3|\right)\Big|_1^2$$

$$= \frac{1}{2}\ln\left|\frac{x+1}{x+3}\right|\Big|_1^2 = \frac{1}{2}\left(\ln\left(\frac{3}{5}\right) - \ln\left(\frac{2}{4}\right)\right)$$

$$= \frac{1}{2}\ln\left(\frac{3}{5}\times\frac{4}{2}\right) = \frac{1}{2}\ln\left(\frac{6}{5}\right)$$

8. (8 points) In order to evaluate the integral $\int \frac{4x^3-5x^2+8x-10}{(x+2)(x-2)^3}\,dx$, how do you express the integrand as sum of partial fractions?

A. $\boxed{\frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3}}$

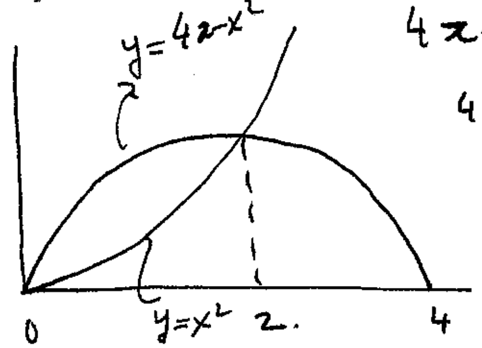
B. $\frac{A}{x+2} + \frac{B}{x-2}$

C. $\frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

D. $\frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{D}{(x+2)^2}$

E. $\frac{A}{x+2} + \frac{B}{(x-2)^3}$

9. (8 points) The area of the region of the plane bounded by the curves $y = 4x - x^2$ and $y = x^2$ is equal to $\frac{8}{3}$. The y-coordinate of its centroid is equal to



$4x - x^2 = x^2$

$4x = 2x^2$

$x = 0$

$x = 2.$

A. 3

B. $\frac{3}{2}$

C. $\frac{4}{3}$

D. $\frac{3}{4}$

E. 2

$$\bar{y} = \frac{1}{A}\int_0^2 \frac{1}{2}\left[(4x-x^2)^2 - x^4\right]dx =$$

$A = 8/3$

$$= \frac{3}{16}\int_0^2 (4x-2x^2)\,4x\,dx = \frac{3}{2}\int_0^2 (2x^2 - x^3)\,dx$$

$$= 3/2 \cdot \left(\frac{2}{3}x^3 - \frac{x^4}{4}\right)\Big|_0^2 = \frac{3}{2}\cdot\left(\frac{16}{3} - \frac{16}{4}\right) = 24.\left(\frac{1}{3} - \frac{1}{4}\right)$$

$$= 2.$$

10. (8 points) Compute $\lim_{n\to\infty}\left(\frac{n^2+2n-4}{2n-5} - \frac{n}{2}\right)$

A. $\frac{3}{4}$

B. $\frac{9}{4}$

C. $\frac{7}{4}$

D. 0

E. ∞

$$= \lim_{n\to\infty} \frac{\cancel{2n^2} + 4n - 8 - \cancel{2n^2} + 5n}{2(2n-5)}$$

$$= \lim_{n\to\infty} \frac{9n-8}{4n-10} = 9/4$$

11. (8 points) The sum of the series $\sum_{n=1}^{\infty} \frac{1+2^n}{5^n}$ is equal to

A. $\frac{6}{5}$

B. $\frac{9}{5}$

C. $\boxed{\frac{11}{12}}$

D. $\frac{12}{5}$

E. $\frac{8}{9}$

$$\sum_{n=1}^{\infty} \frac{1+2^n}{5^n} = \sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n + \sum_{n=1}^{\infty} \left(\frac{2}{5}\right)^n$$

$$= \frac{1/5}{1-1/5} + \frac{2/5}{1-2/5} = \frac{1/5}{4/5} + \frac{2/5}{3/5}.$$

$$= 1/4 + 2/3 = \frac{3+8}{12} = \frac{11}{12}$$

12. (4 points) The series $\sum_{n=1}^{\infty} \frac{4n+5}{n^2}$ converges by the integral test

A. True

B. False

$\frac{4n+5}{n^2} \sim \frac{1}{n}$ the series diverges.

$$\int_1^{\infty} \frac{4x+5}{x^2}\,dx = \int_1^{\infty} \frac{4}{x} + \int \frac{5}{x^2}\,dx,$$ the first integral diverges!

13. (4 points) If the series $\sum_{k=1{,}000{,}000{,}000}^{\infty} a_k$ diverges, one cannot say whether the series $\sum_{k=1}^{\infty} a_k$ converges or diverges.

A. True

B. False

The difference of the two sums is finite. If one diverges, so does the other one.

14. (4 points) The series $\sum_{n=1}^{\infty} \cos(\frac{1}{n})$ converges.

A. True

B. False

$$\lim_{n\to\infty} \cos(1/n) = 1 \neq 0$$

the series diverges.

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TEST NUMBER 02

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STUDENT SIGNATURE: ____________________

STUDENT ID NUMBER: ____________________

SECTION NUMBER AND RECITATION INSTRUCTOR: ____________________

1. **(8 points)** Compute $\int_0^1 xe^{4x}\,dx$.

A. $\dfrac{e^4+1}{8}$

$\boxed{\text{B. } \dfrac{3e^4+1}{16}}$

C. $\dfrac{3e^4+5}{16}$

D. $\dfrac{5e^4-2}{8}$

E. $\dfrac{3e^4-4}{16}$

$x = u, \quad e^{4x}dx = dv$

$du = dx; \quad v = \frac{1}{4}e^{4x}.$

$$\int x e^{4x} dx = \frac{1}{4} x e^{4x} - \frac{1}{4}\int e^{4x} dx$$

$$= \frac{1}{4} x e^{4x} - \frac{1}{16} e^{4x}.$$

$$\int_0^1 x e^{4x} dx = \left(\frac{1}{4} x e^{4x} - \frac{1}{16} e^{4x}\right)\Big|_0^1$$

$$= \left(\frac{1}{4} e^{4} - \frac{1}{16} e^{4}\right) - \left(-\frac{1}{16}\right) = \frac{3e^4+1}{16}.$$

2. **(8 points)** Compute $\int_0^{\frac{\pi}{6}} 5(\sec^4 x)(1-\tan^2 x)\,dx$.

A. $44\sqrt{3}$

B. $40\sqrt{3}$

C. $\dfrac{40}{\sqrt{3}}$

D. $\dfrac{40}{9\sqrt{3}}$

$\boxed{\text{E. } \dfrac{44}{9\sqrt{3}}}$

$\sec^2 x = 1 + \tan^2 x$

$$= \int_0^{\pi/6} 5 \sec^2 x . (1+\tan^2 x)(1-\tan^2 x)\, dx$$

$\tan x = u, \quad du = \sec^2 x$

$$= 5\int_0^{1/\sqrt{3}} (1+u^2)(1-u^2)\, du$$

$$=5\int_0^{1/\sqrt{3}} (1-u^4)\, du = 5\left(u - \frac{u^5}{5}\right)\Big|_0^{1/\sqrt{3}}$$

$$= 5\left(\frac{1}{\sqrt{3}} - \frac{1}{5}\cdot\left(\frac{1}{\sqrt{3}}\right)^5\right) = 5\left(\frac{1}{\sqrt{3}} - \frac{1}{5}\cdot\frac{1}{9}\frac{1}{\sqrt{3}}\right)$$

$$= \frac{5}{\sqrt{3}}\left(1 - \frac{1}{45}\right) = 5\cdot\frac{44}{45\sqrt{3}} = \frac{44}{9\sqrt{3}}$$

3. **(8 points)** If we use a trigonometric substitution to evaluate $\int \frac{x^3}{\sqrt{x^2+9}}\,dx$ the integral becomes

A. $\int 9\sec^3\theta\tan\theta\,d\theta$

$\boxed{\text{B. } \int 27\sec\theta\,\tan^3\theta\,d\theta}$

C. $\int 27\sec^4\theta\,d\theta$

D. $\int 27\sec^2\theta\,\tan^3\theta\,d\theta$

E. $\int 9\tan^3\theta\,\sec\theta\,d\theta$

$x = 3\tan\theta,\quad dx = 3\sec^2\theta\,d\theta$

$\sqrt{x^2+9} = \sqrt{9(\tan^2\theta+1)} = 3\sec\theta.$

$\int \frac{x^3\,dx}{\sqrt{x^2+9}} = \int \frac{27\tan^3\theta \cdot 3\sec^2\theta\,d\theta}{3\sec\theta}$

$= 27\int \tan^3\theta \cdot \sec\theta\,d\theta.$

4. **(8 points)** Compute $\int_0^{\frac{\sqrt{2}}{2}} \frac{x^3}{(1-x^2)^{\frac{3}{2}}}\,dx.$

A. $\frac{6\sqrt{2}-5}{2}$

B. $\frac{5\sqrt{2}-3}{2}$

$\boxed{\text{C. } \frac{3\sqrt{2}-4}{2}}$

D. $\frac{2\sqrt{2}-1}{2}$

E. $\frac{8\sqrt{2}-9}{2}$

$x = \sin\theta.$

$dx = \cos\theta\,d\theta$

$(1-x^2)^{3/2} = (1-\sin^2\theta)^{3/2}$

$= (\cos^2\theta)^{3/2} = \cos^3\theta$

$= \int_0^{\pi/4} \frac{\sin^3\theta}{\cos^3\theta} \cdot \cos\theta\,d\theta$

$= \int_0^{\pi/4} \sin\theta \cdot \frac{\sin^2\theta}{\cos^2\theta}\,d\theta = \int_0^{\pi/4} \sin\theta\left(\frac{1-\cos^2\theta}{\cos^2\theta}\right)d\theta$

$\cos\theta = u \quad du = -\sin\theta\,d\theta$

$= \int_{1/\sqrt{2}}^{1} (u^{-2}-1)\,du = -(u+u^{-1})\Big|_{1/\sqrt{2}}^{1}$

$= -\left[2 - \left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)\right] = -\left[\frac{2\sqrt{2}-2-1}{\sqrt{2}}\right] = \frac{3-2\sqrt{2}}{\sqrt{2}} =$

$= \frac{3\sqrt{2}-4}{2}.$

5. (8 points) Find the length of the curve $y = \frac{5}{12}\left(x^{6/5} - \frac{3}{2}x^{4/5}\right)$, for $0 \le x \le 1$. The arc-length formula holds in this case.

A. $\frac{8}{3}$

$$y' = \frac{5}{12}\cdot\frac{6}{5}x^{1/5} - \frac{5}{12}\cdot\frac{3}{2}\cdot\frac{4}{5}x^{-1/5}$$

B. $\boxed{\frac{25}{24}}$

$$y' = \frac{1}{2}x^{1/5} - \frac{1}{2}x^{-1/5}$$

C. $\frac{32}{7}$

$$1+(y')^2 = 1 + \frac{1}{4}\left(x^{2/5} + x^{-2/5} - 2\right)$$

D. $\frac{27}{24}$

$$= \frac{1}{4}\left(x^{2/5} + x^{-2/5} + 2\right) = \frac{1}{4}\left(x^{1/5} + x^{-1/5}\right)^2$$

E. $\frac{15}{32}$

$$L = \int_0^1 \sqrt{1+(y')^2}\,dx = \int_0^1 \frac{1}{2}\left(x^{1/5} + x^{-1/5}\right)dx$$

$$= \frac{1}{2}\cdot\frac{5}{6}x^{6/5} + \frac{1}{2}\cdot\frac{5}{4}x^{4/5}\Big|_0^1 = \frac{5}{12}x^{6/5} + \frac{5}{8}x^{4/5}\Big|_0^1$$

$$= \frac{5}{12} + \frac{5}{8} = 5\ \frac{5}{24} = \frac{25}{24}$$

6. (8 points) Which of the following improper integrals are convergent?

I. $\int_{-1}^{\infty} \frac{1}{x+2}\,dx$ F   II. $\int_0^{\infty} \frac{1}{4+x^2}\,dx$ T   III. $\int_0^{\infty} e^{-x}dx$ T

A. I and II only

$$\int_{-1}^{\infty} \frac{dx}{x+2} = \int_1^{\infty} \frac{du}{u} \quad \text{diverges}$$

$u = x+2.$

B. $\boxed{\text{II and III only}}$

C. II only

$$0 \le \int_0^{\infty} \frac{dx}{4+x^2} \le \int_0^{\infty} \frac{dx}{1+x^2} = \pi/2$$

converges.

D. None of them

E. All of them

$$\int_0^{\infty} e^{-x}dx = -e^{-x}\Big|_0^{\infty} = 1.$$

7. (8 points) Compute the integral $\int_0^1 \frac{1}{x^2+7x+6}\,dx$.

$x^2+7x+6 = (x+1)(x+6)$

A. $\frac{1}{5}\ln(\frac{1}{9})$

B. $\frac{1}{5}\ln(\frac{9}{7})$

C. $\frac{1}{5}\ln(\frac{8}{5})$

D. $\frac{1}{5}\ln(\frac{12}{7})$

E. $\frac{1}{5}\ln(\frac{8}{9})$

$$\frac{1}{x^2+7x+6} = \frac{1}{5}\left(\frac{1}{x+1} - \frac{1}{x+6}\right).$$

$$\int_0^1 \frac{dx}{x^2+7x+6}\,dx = \frac{1}{5}\int_0^1 \left(\frac{1}{x+1} - \frac{1}{x+6}\right) dx$$

$$= \frac{1}{5}\left(\ln|x+1| - \ln|x+6|\right)\Big|_0^1 =$$

$$= \frac{1}{5}\ln\left|\frac{x+1}{x+6}\right|\Big|_0^1 = \frac{1}{5}\left[\ln\left(\frac{2}{7}\right) - \ln\left(\frac{1}{6}\right)\right]$$

$$= \frac{1}{5}\ln\left(\frac{2}{7}\times\frac{6}{1}\right) = \frac{1}{5}\ln\left(\frac{12}{7}\right)$$

8. (8 points) In order to evaluate the integral $\int \frac{4x^3-5x^2+8x-10}{(x+2)^3(x-2)}\,dx$, how do you express the integrand as sum of partial fractions?

A. $\frac{A}{x+2} + \frac{B}{x-2}$

B. $\frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{(x+2)^2}$

C. $\frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{(x+2)^3}$

D. $\frac{A}{x-2} + \frac{B}{(x+2)^3}$

E. $\frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{(x+2)^2} + \frac{D}{(x+2)^3}$

9. (8 points) The area of the region of the plane bounded by the curves $y = 4x - x^2$ and $y = x^2$ is equal to $\frac{8}{3}$. The x-coordinate of its centroid is equal to

$\boxed{\text{A. } 1}$

B. $\frac{3}{4}$

C. $\frac{2}{3}$

D. $\frac{1}{2}$

E. $\frac{3}{2}$

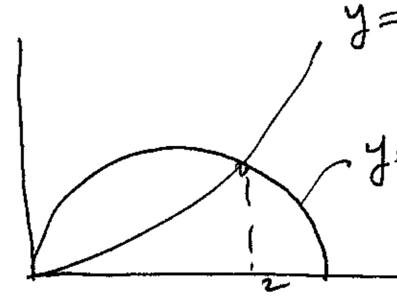
$$\bar{x} = \frac{1}{A} \int_0^2 x(4x - x^2 - x^2)\,dx$$

$A = 8/3$

$$\bar{x} = \frac{3}{8} \cdot \int_0^2 (4x - 2x^2)\,dx$$

$$= \frac{3}{8} \cdot \left(2x^2 - \frac{2x^3}{3}\right)\Big|_0^2$$

$$= \frac{3}{8} \cdot \left(8 - \frac{16}{3}\right) = \frac{3}{8} \cdot \frac{24-16}{3} = 1.$$



$x^2 = 4x - x^2$

$4x = 2x^2$

$x = 0,\ x = 2$

10. (8 points) Compute $\lim_{n\to\infty} \left(\frac{n^2 + 2n - 4}{n - 5} - n\right) = \lim_{n\to\infty} \left(\frac{\cancel{n^2} + 2n - 4 - \cancel{n^2} + 5n}{n - 5}\right)$

A. 2

B. 9

$\boxed{\text{C. } 7}$

D. 0

E. ∞

$$= \lim_{n\to\infty} \left(\frac{7n - 4}{n - 5}\right) = 7$$

11. **(8 points)** The sum of the series $\sum_{n=1}^{\infty} \frac{1+4^n}{7^n}$ is equal to

A. $\frac{6}{5}$

B. $\frac{3}{2}$

C. $\frac{9}{5}$

D. $\frac{12}{5}$

E. $\frac{19}{20}$

$$\sum_{n=1}^{\infty} \frac{1+4^n}{7^n} = \sum_{n=1}^{\infty} \frac{1}{7^n} + \sum_{n=1}^{\infty} \left(\frac{4}{7}\right)^n$$

$$= \frac{1/7}{1-1/7} + \frac{4/7}{1-4/7} = \frac{1/7}{6/7} + \frac{4/7}{3/7}$$

$$= 1/6 + 4/3 = \frac{1+8}{6} = 9/6 = 3/2$$

12. **(4 points)** The series $\sum_{n=1}^{\infty} \frac{4n+5}{n^3}$ converges by the integral test

A. True

B. False

$$\int_1^{\infty} \frac{4x+5}{x^3}\,dx \leq \int_1^{\infty} \frac{4}{x^2}\,dx + \int_1^{\infty} 5/x^3\,dx < \infty$$

13. **(4 points)** If a series $\sum_{k=1,000,000,000}^{\infty} a_k$ converges, then the series $\sum_{k=1}^{\infty} a_k$ converges.

A. Always true

B. Not always true, it depends on a_k.

The difference between the sums is finite. If one converges, so does the other one.

14. **(4 points)** The series $\sum_{n=1} e^{\frac{1}{n}}$ diverges.

A. True

B. False

$$\lim_{n\to\infty} e^{1/n} = 1 \neq 0$$

the series diverges