NAME GRADING KEY

STUDENT ID ______

RECITATION INSTRUCTOR ______

RECITATION TIME ______

| | |
|---|---|
| Page 1 | /14 |
| Page 2 | /30 |
| Page 3 | /28 |
| Page 4 | /28 |
| TOTAL | /100 |

DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

---

(2) 1. Find an expression for the function whose graph is the top half of the circle $(x-1)^2 + y^2 = 1$.

$y = \sqrt{1-(x-1)^2}$

NPC

$f(x) = \sqrt{1-(x-1)^2}$ $\boxed{2}$

(4) 2. Find all values of $x$ in the interval $[0, 2\pi]$ that satisfy the equation $2\cos x - 1 = 0$.

$\cos x = \frac{1}{2} \rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{3}$ (2), $\frac{5\pi}{3}$ (2) $\boxed{4}$

−1 pt. for each extra answer

(8) 3. Let $f(x) = 2x + c$ and $g(x) = 3x + c^2$ where $c$ is a constant.

(a) Find the composite functions $f \circ g$ and $g \circ f$.

$(f\circ g)(x) = f(g(x)) = 2g(x) + c$
$= 2(3x + c^2) + c$

$(g\circ f)(x) = g(f(x)) = 3f(x) + c^2$
$= 3(2x + c) + c^2$

$(f \circ g)(x) = 6x + 2c^2 + c$ (3) NPC

$(g \circ f)(x) = 6x + c^2 + 3c$ (3) NPC

(b) Find the value(s) of $c$ for which $f \circ g = g \circ f$.

$6x + 2c^2 + c = 6x + c^2 + 3c$
$c^2 - 2c = 0 \rightarrow c = 0, 2$

$c = 0$ (1), $2$ (1) $\boxed{8}$

Name: ______________________________

(10) 4. Write the equation of the graph that results by

2 pts each NPC

(a) shifting the graph of $y = e^x$ 2 units downward

$\boxed{y = e^x - 2}$

(b) shifting the graph of $y = e^x$ 2 units to the right

$\boxed{y = e^{x-2}}$

(c) reflecting the graph of $y = e^x$ about the $x$–axis

$\boxed{y = -e^x}$

(d) reflecting the graph of $y = e^x$ about the $y$–axis

$\boxed{y = e^{-x}}$

(e) reflecting the graph of $y = e^x$ about the $x$–axis and then about the $y$–axis

$\boxed{y = -e^{-x}}$

$\boxed{10}$

(10) 5. Find a formula for the inverse $f^{-1}$ of the function $f(x) = \dfrac{1+3x}{5-2x}$ and give the domain of $f^{-1}$.

$y = \dfrac{1+3x}{5-2x}$ 2 $\qquad y = f(x) \iff x = f^{-1}(y)$

$5y - 2xy = 1 + 3x$

$5y - 1 = x(3+2y)$

$x = \dfrac{5y-1}{3+2y} \quad \therefore f^{-1}(y) = \dfrac{5y-1}{3+2y}$

4

$f^{-1}(x) = \dfrac{5x-1}{3+2x}$ 2

domain of $f^{-1} = (-\infty, -\frac{3}{2}) \cup (-\frac{3}{2}, \infty)$

or all $x \neq -\frac{3}{2}$ 2

$\boxed{10}$

(10) 6. For each of the functions below find the value of $k$ such that the function is continuous in $\mathbb{R}$, or state that there is "no such $k$".

(a) $f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \neq 3 \\ k & \text{if } x = 3 \end{cases}$

continuous for all $x \neq 3$

cont. at $x = 3$?

$\lim\limits_{x \to 3} \dfrac{x^2-9}{x-3} = \lim\limits_{x \to 3} \dfrac{(x+3)(x-3)}{x-3} = 6$

cont. if $f(3) = 6$

$\boxed{k = 6}$ NPC $\boxed{5}$

(b) $f(x) = \begin{cases} \dfrac{x^2+5x+4}{x-1} & \text{if } x \neq 1 \\ k & \text{if } x = 1 \end{cases}$

continuous for all $x \neq 1$

cont at $x = 1$?

$\lim\limits_{x \to 1^+} \dfrac{x^2+5x+4}{x-1} = +\infty$ $\therefore$ not cont at $x = 1$

$\boxed{\text{no such } k}$ NPC $\boxed{5}$

Name: ______________________________

(18) 7. For each of the following fill in the boxes below with a finite number, or one of the symbols $\infty$, $-\infty$, or DNE (does not exist). It is not necessary to give reasons for your answers.

3 pts for each correct answer (NPC)

(a) $\lim_{x\to 5^+} \frac{6}{x-5} = \infty$

$\boxed{\infty}$

(b) $\lim_{x\to 0} \frac{\sin 2x}{x} = \lim_{t\to 0} \frac{\sin t}{\frac{t}{2}} = 2 \lim_{t\to 0} \frac{\sin t}{t} = 2$

$t = 2x$

$\boxed{2}$

(c) $\lim_{x\to 0} \frac{x}{|x|} =$

$\lim_{x\to 0^+} \frac{x}{|x|} = \lim_{x\to 0^+} 1 = 1$

$\lim_{x\to 0^-} \frac{x}{|x|} = \lim_{x\to 0^-} (-1) = -1$

$\boxed{\text{DNE}}$

(d) $\lim_{x\to 0^-} \csc x = \lim_{x\to 0^-} \frac{1}{\sin x} = -\infty$

$\sin x < 0$ for $x < 0$ and close to 0

and $\sin x \to 0$ as $x \to 0^-$

$\boxed{-\infty}$

(e) $\lim_{x\to 100} \frac{x-100}{\sqrt{x}-10} = \lim_{x\to 100} \frac{(\sqrt{x}-10)(\sqrt{x}+10)}{\sqrt{x}-10}$

$= \lim_{x\to 100} (\sqrt{x}+10) = 20$

$\boxed{20}$

(f) $\lim_{x\to 0^+} \ln(e^x - 1) =$

as $x \to 0^+$, $e^x - 1 > 0$ and $e^x - 1 \to 0$

$\therefore \ln(e^x - 1) \to -\infty$

$\boxed{-\infty}$

$\boxed{18}$

(10) 8. Find the derivative of $f(x) = \frac{1}{x^2}$ using the definition of the derivative:

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$. (0 credit for using the formula for the derivative).

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \quad 4$$

$$= \lim_{h\to 0} \frac{x^2 - (x+h)^2}{h(x+h)^2 x^2} = \lim_{h\to 0} \frac{x^2 - x^2 - 2hx - h^2}{h(x+h)^2 x^2}$$

$$= \underbrace{\lim_{h\to 0} \frac{-2x-h}{(x+h)^2 x^2}}_{4} = \frac{-2x}{x^4} = -\frac{2}{x^3}$$

or 2

$\boxed{10}$

Name: ______________________________

(10) 9. Find the values of the constants $a$ and $b$ so that the curve $y = ax^2 + bx$ passes through the point $(1, 1)$ and the tangent line to the curve at $(1, 1)$ is parallel to the line $y = 3x$.

(1,1) is on the curve $\longrightarrow$ $1 = a + b$ 4

$\frac{dy}{dx} = 2ax + b$, when $x = 1$ $\frac{dy}{dx} = 3$ $\therefore$ $3 = 2a + b$ 4

$\left.\begin{aligned} a + b &= 1 \\ 2a + b &= 3 \end{aligned}\right\} \rightarrow a = 2,\ b = -1$

$a = 2$ 1, $b = -1$ 1

10

(12) 10. Find the derivatives of the following functions. (It is not necessary to simplify).

NPC

(a) $y = x\sqrt{x} - \dfrac{1}{x^2\sqrt{x}} = x^{3/2} - x^{-5/2}$

$\frac{dy}{dx} = \frac{3}{2} x^{1/2} + \frac{5}{2} x^{-7/2}$

$\frac{dy}{dx} = \frac{3}{2} x^{\frac{1}{2}} + \frac{5}{2} x^{-\frac{7}{2}}$

4

(b) $f(x) = e^x \tan x$

$f'(x) = e^x \sec^2 x + e^x \tan x$

$f'(x) = e^x \sec^2 x + e^x \tan x$

4

(c) $g(x) = \dfrac{\cos x}{1 + \sin x}$

$g'(x) = \dfrac{(1+\sin x)(-\sin x) - \cos x \cos x}{(1+\sin x)^2}$

4

$g'(x) = \dfrac{(1+\sin x)(-\sin x) - \cos x \cos x}{(1+\sin x)^2}$

$= \dfrac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2} = -\dfrac{1+\sin x}{(1+\sin x)^2} = -\dfrac{1}{1+\sin x}$

-1 pt if initial answer is correct and a mistake occurs in simplifying

(6) 11. Evaluate the following:

NPC

(a) $\sin(\pi e^{-\ln 2}) = \sin\left(\pi \frac{1}{e^{\ln 2}}\right) = \sin \frac{\pi}{2} = 1$

1

3

(b) $\tan(\pi \ln e^{\frac{3}{4}}) = \tan\left(\pi \frac{3}{4}\right) = -1$

$-1$

3