NAME GRADING KEY

10-digit PUID ____________

RECITATION INSTRUCTOR ____________

RECITATION TIME ____________

Page 1	/12
Page 2	/30
Page 3	/28
Page 4	/30
TOTAL	/100

DIRECTIONS

1. Write your name, 10–digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

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(6) 1. Find a formula for the inverse of $f(x) = \sqrt{7-2x}$.

$x = f^{-1}(y) \Longleftrightarrow y = f(x)$

$y = \sqrt{7-2x} \rightarrow y^2 = 7-2x$

$x = \frac{7-y^2}{2}$

$\therefore f^{-1}(y) = \frac{7-y^2}{2}$

$f^{-1}(x) = \frac{7-x^2}{2}$

-3pts for $\boxed{f^{-1}(x) = \frac{7-y^2}{2}}$

$\boxed{f^{-1}(x) = \frac{7-x^2}{2}}$ $\boxed{6}$

(6) 2. Let $f(x) = 2x + c$ and $g(x) = 3x + c^2$. Find a nonzero constant c such that

$$(f \circ g)(x) = (g \circ f)(x).$$

$(f\circ g)(x) = f(g(x)) = f(3x+c^2) = 2(3x+c^2)+c = 6x+2c^2+c$

$(g\circ f)(x) = g(f(x)) = g(2x+c) = 3(2x+c)+c^2 = 6x+3c+c^2$

$6x+2c^2+c = 6x+3c+c^2$ or 3

$2c^2+c = 3c+c^2$

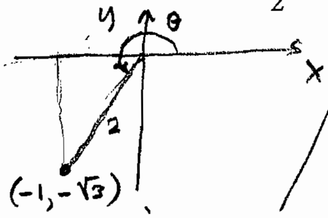
$c^2-2c = 0 \rightarrow c = 0, 2$

-1 pt for $\boxed{c = 0, 2}$

3

$\boxed{c = 2}$ $\boxed{6}$

(6) 3. If $\cos\theta = -\frac{1}{2}$ and $\pi < \theta < \frac{3\pi}{2}$, find the following:



or $\sin^2\theta + \cos^2\theta = 1$

$\sin\theta = \pm\sqrt{1-\cos^2\theta}$

$\ominus$ because $\pi < \theta < \frac{3\pi}{2}$

$= -\sqrt{1-\frac{1}{4}} = -\frac{\sqrt{3}}{2}$

$\sin\theta =$ $-\frac{\sqrt{3}}{2}$ ③
$\tan\theta =$ $\sqrt{3}$ ③

$\boxed{6}$

(6) 4. Solve the equation $3x + 2|x - 3| = 7$.

If $x \geq 3$; $3x + 2(x-3) = 7 \to 5x = 13 \to x = \frac{13}{5}$ ← not a solution because $\frac{13}{5} < 3$

If $x < 3$: $3x + 2[-(x-3)] = 7 \to x = 1$

-1 pt for $\boxed{x = 1, \frac{13}{5}}$

$x =$ 1

$\boxed{6}$

(6) 5. Find the equations of the vertical and horizontal asymptotes of the function $f(x) = \dfrac{1-2x}{x+6}$.

$\lim\limits_{x\to(-6)^+} \frac{1-2x}{x+6} = \infty$ $\therefore$ $x = -6$ V.A.

$\lim\limits_{x\to\infty} \frac{1-2x}{x+6} = \lim\limits_{x\to\infty} \frac{\frac{1}{x}-2}{1+\frac{6}{x}} = -2$ $\therefore$ $y = -2$ H.A.

Vertical asymptotes	Horizontal asymptotes
$x = -6$ ③	$y = -2$ ③

$\boxed{6}$

(6) 6. (a) Complete the definition: The function f is continuous at a if $\lim\limits_{x\to a} f(x) = f(a)$ ②

(b) For what value of a is the function f continuous?

$$f(x) = \begin{cases} ax^2, & \text{if } x \leq 1 \\ \sqrt{x} - a, & \text{if } x > 1 \end{cases}$$

$\lim\limits_{x\to1^-} f(x) = \lim\limits_{x\to1^-} ax^2 = a$, $\lim\limits_{x\to1^+} f(x) = \lim\limits_{x\to1^+} (\sqrt{x} - a) = 1 - a$

$\lim\limits_{x\to1^-} f(x) = \lim\limits_{x\to1^+} f(x)$ ②

$a = 1 - a \to a = \frac{1}{2}$

$a =$ $\frac{1}{2}$ ②

$\boxed{6}$

(6) 7. Find an equation of the tangent line to the curve $y = 3x^2 - 5x$ at the point $(2, 2)$.

$\frac{dy}{dx} = 6x - 5$ ② $\left.\frac{dy}{dx}\right|_{x=2} = 7$ ②

$y - 2 = 7(x-2)$
or $y = 7x - 12$ ②

$\boxed{6}$

(16) 8. For each of the following, fill in the boxes below with a finite number, or one of the symbols $+\infty, -\infty$, or DNE (does not exist). It is not necessary to give reasons for your answers.

NPC

(a) $\lim_{x\to\infty} x(\sqrt{4x^2+1}-2x) =$ $\boxed{\frac{1}{4}}$ (4)

$= \lim_{x\to\infty} x(\sqrt{4x^2+1}-2x)\frac{\sqrt{4x^2+1}+2x}{\sqrt{4x^2+1}+2x} = \lim_{x\to\infty} \frac{x(4x^2+1-4x^2)}{\sqrt{4x^2+1}+2x} = \lim_{x\to\infty} \frac{x}{\sqrt{4x^2+1}+2x} =$

$= \lim_{x\to\infty} \frac{1}{\sqrt{4+\frac{1}{x^2}}+2} = \frac{1}{2+2} = \frac{1}{4}$

(b) $\lim_{x\to1} \frac{x-2}{(x-1)^2} = -\infty$ $\boxed{-\infty}$ (4)

(c) $\lim_{x\to2^+} \frac{|2-x|}{2-x} = \lim_{x\to2^+} \frac{-(2-x)}{2-x} = -1$ $\boxed{-1}$ (4)

(d) $\lim_{x\to\infty} \frac{x+x^3+3x^5}{1-x^2+x^4} = \infty$ $\boxed{\infty}$ (4)

(4) 9. Simplify $\ln(\ln e^e)$.

$= \ln(e\ln e) = \ln(e\cdot 1) = \ln e = 1$

$\boxed{\ln(\ln e^e) = 1}$ 4

(4) 10. Solve $\ln(x+1)^2 = 2$ for x.

$e^{\ln(x+1)^2} = e^2$

$(x+1)^2 = e^2$

$x+1 = \pm e \rightarrow x = e-1, -e-1$

$\boxed{x = e-1, \; x = -e-1}$ 4

(4) 11. If $\lim_{x\to2} \frac{f(x)-5}{x-2} = 4$, find $\lim_{x\to2} f(x)$.

$\lim_{x\to2} [f(x)-5] = \lim_{x\to2} \frac{f(x)-5}{x-2}(x-2) = 4\cdot 0 = 0$

$\therefore \lim_{x\to2} f(x) = 5$

$\boxed{\lim_{x\to2} f(x) = 5}$ (4)

(10) 12. Find the derivative of the function $f(x) = \dfrac{2}{x+3}$ using the definition of the derivative: $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$. (0 credit for using a formula for the derivative).

$$f'(x) = \lim_{h \to 0} \frac{\frac{2}{(x+h)+3} - \frac{2}{x+3}}{h} \quad 4 \quad = \lim_{h\to 0} \frac{2(x+3) - 2(x+h+3)}{h(x+h+3)(x+3)}$$

$$= \lim_{h \to 0} \frac{-2\cancel{h}}{\cancel{h}(x+h+3)(x+3)} \quad 3 \quad = \frac{-2}{(x+3)^2} \quad 3$$

-1 pt for early ommission of $\lim\limits_{h\to 0}$

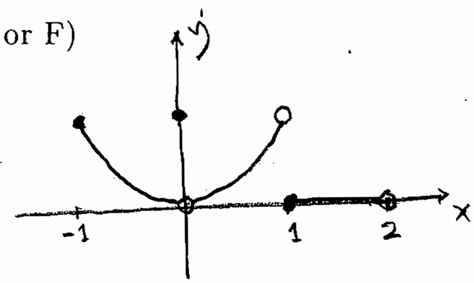
10

(6) 13. Which of the following statements about the function

$$f(x) = \begin{cases} x^2, & \text{if } -1 \le x < 0 \\ 1, & \text{if } x = 0 \\ x^2, & \text{if } 0 < x < 1 \\ 0, & \text{if } 1 \le x < 2 \end{cases}$$

NPC

are true and which are false? (Circle T or F)



(a) $\lim\limits_{x \to 1^-} f(x) = 0$ T F 2

(b) $\lim\limits_{x \to 0} f(x) = 1$ T F 2

(c) $\lim\limits_{x \to (-1)^+} f(x) = 1$ T F 2

(4) 14. For what value(s) of x does the graph of $f(x) = 3x^2 + x + 7$ have a horizontal tangent?

$f'(x) = 6x + 1$ 2

$f'(x) = 0$: $6x + 1 = 0$ 1 $\rightarrow x = -\frac{1}{6}$ 1

$x = -\dfrac{1}{6}$

4

(10) 15. Find the derivatives of the following functions. (It is not necessary to simplify).

(a) $y = (\tan x)(x^3 + 2)$.

$\dfrac{dy}{dx} = (\tan x)(3x^2) + (x^3+2)\sec^2 x$ 5

(b) $f(x) = \dfrac{e^x}{1+x}$.

$\dfrac{dy}{dx} = \dfrac{(1+x)e^x - e^x(1)}{(1+x)^2}$ 5