NAME GRADING KEY

STUDENT ID

RECITATION INSTRUCTOR

RECITATION TIME

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TOTAL	/100

DIRECTIONS

1. Write your name, 10–digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes, calculators or any electronic devices may be used on this exam.

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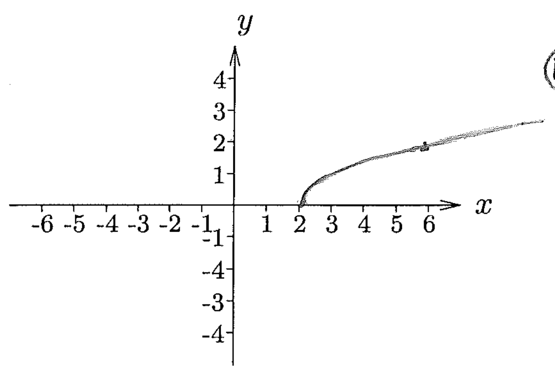
(7) 1. Find the domain and sketch the graph of the function $g(x) = \sqrt{x-2}$.



5

2

Domain : $[2, \infty)$ or $x \geq 2$

7

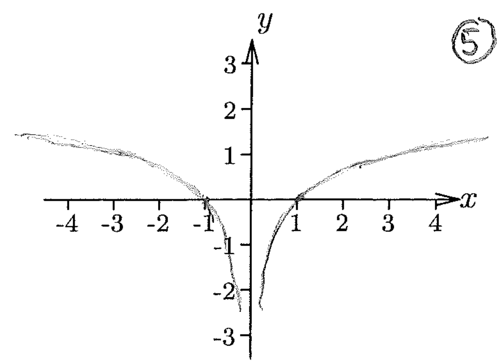
(7) 2. (a) Is the function $f(x) = \ln|x|$ even, odd, or neither?

$\ln|-x| = \ln|x|$

2

even

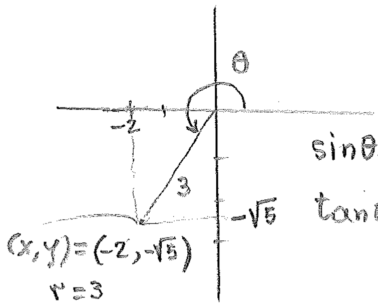
(b) Make a rough sketch of the graph of $y = \ln|x|$



5

7

(8) 3. If $\cos\theta = -\frac{2}{3}$, $\pi < \theta < \frac{3\pi}{2}$, find the following:



$\sin\theta = -\frac{\sqrt{5}}{3}$

$\tan\theta = \frac{\sqrt{5}}{2}$

or $\sin^2\theta = 1-\cos^2\theta$

$= 1 - \frac{4}{9} = \frac{5}{9}$

$\therefore \sin\theta = \pm\frac{\sqrt{5}}{3}$

$\sin\theta = -\frac{\sqrt{5}}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{\sqrt{5}}{3}}{-\frac{2}{3}} = \frac{\sqrt{5}}{2}$

NPC

$\sin\theta = -\frac{\sqrt{5}}{3}$ (4)

$\tan\theta = \frac{\sqrt{5}}{2}$ (4)

8

(6) 4. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation $\sin 2x = \sqrt{3}\ \cos x$.

$2\sin x\cos x = \sqrt{3}\cos x$ (2)

$\cos x(2\sin x - \sqrt{3}) = 0$

$\cos x = 0 \rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

or

$\sin x = \frac{\sqrt{3}}{2} \rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}$

−1pt for each answer beyond 4

(1) (1) (1) (1)

$\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2}$

6

(10) 5. If $f(x) = 2 - e^x$, find the following:

(a) A formula for the inverse function $f^{-1}(x)$.

$y = f(x) \Longleftrightarrow x = f^{-1}(y)$

$y = 2 - e^x$

$e^x = 2 - y$

$x = \ln(2-y)$

$\therefore f^{-1}(y) = \ln(2-y)$

$f^{-1}(x) = \ln(2-x)$

−3pts for $f^{-1}(x) = \ln(2-y)$

(6) $f^{-1}(x) = \ln(2-x)$

(b) The domain of f^{-1}.

(2) $x < 2$ or $(-\infty, 2)$

(c) The range of f^{-1}

(2) all y or $(-\infty, \infty)$

10

(6) 6. Using a theorem about continuous functions, we can conclude that the equation $x^4 + x - 3 = 0$ has a root in one of the following intervals:

A. $(-1, 0)$ B. $(0, 1)$ (4) C. $(1, 2)$ D. $(2, 3)$ E. $(3, 4)$

(a) Circle the letter of that interval.

$f(x) = x^4 + x - 3$

$f(-1) = -3 < 0$, $f(0) = -3 < 0$, $f(1) = -1 < 0$, $f(2) = 15 > 0$, $f(3) = 81 > 0$, $f(4) = 257 > 0$

(b) State the name of the theorem you are using.

Intermediate Value Theorem (2)

6

(12) 7. For each of the following, fill in the boxes below with a finite number or one of the symbols $+\infty$, $-\infty$, or DNE (does not exist). It is not necessary to give reasons for your answers.

2 pts each NPC

(a) $\lim_{x\to 2} \frac{x^2-2x}{x^2-4} = \lim_{x\to 2} \frac{x(x-2)}{(x+2)(x-2)} = \lim_{x\to 2} \frac{x}{x+2} = \frac{2}{4} = \frac{1}{2}$

$\boxed{\frac{1}{2}}$

(b) $\lim_{x\to 1} \frac{2-x}{(x-1)^2} = +\infty$ ($2-x \xrightarrow{+} 1$, $(x-1)^2 \xrightarrow{+} 0$)

$\boxed{+\infty}$

(c) $\lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{3t^2+t}\right) = \lim_{t\to 0} \frac{3t^2+t-t}{t(3t^2+t)} = \lim_{t\to 0} \frac{3t^2}{t^2(3t+1)}$

$= \lim_{t\to 0} \frac{3}{3t+1} = 3$

$\boxed{3}$

(d) $\lim_{x\to 0} x^4 \cos(\frac{1}{x}) =$ $-1 \le \cos(\frac{1}{x}) \le 1$

$-x^4 \le x^4 \cos(\frac{1}{x}) \le x^4$ by squeeze theorem

as $x \to 0$: $-x^4 \to 0$, $x^4 \to 0$ $\therefore$ $x^4\cos(\frac{1}{x}) \to 0$

$\boxed{0}$

(e) $\lim_{x\to(\frac{\pi}{2})^+} e^{\tan x} = \lim_{t\to -\infty} e^t = 0$

Let $t = \tan x$

as $x \to (\frac{\pi}{2})^+$ $t \to -\infty$

$\boxed{0}$

(f) $\lim_{x\to -5} \frac{2x+10}{|x+5|} =$ $\lim_{x\to(-5)^-} \frac{2x+10}{|x+5|} = \lim_{x\to(-5)^-} \frac{2x+10}{-(x+5)} = -2$

$\lim_{x\to(-5)^+} \frac{2x+10}{|x+5|} = \lim_{x\to(-5)^+} \frac{2x+10}{x+5} = 2$

$\boxed{\text{DNE}}$

$\boxed{12}$

(6) 8. Write the equations of the vertical and horizontal asymptotes, if any, of the graph of $y = \frac{2x+1}{x-2}$.

3pts each NPC

$\lim_{x\to 2^+} \frac{2x+1}{x-2} = \infty$ ($2x+1 \xrightarrow{+} 5$, $x-2 \xrightarrow{+} 0$) $\therefore$ $x=2$ is a V.A.

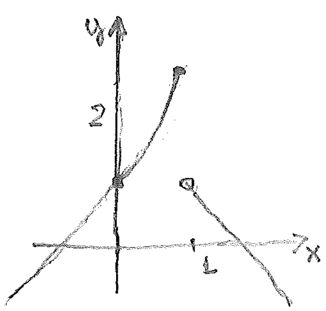
$\lim_{x\to \pm\infty} \frac{2x+1}{x-2} = \lim_{x\to\pm\infty} \frac{2+\frac{1}{x}}{1-\frac{2}{x}} = 2$ $\therefore$ $y=2$ is a H.A.

Vertical asymptotes
$x=2$

Horizontal asymptotes
$y=2$

$\boxed{6}$

(6) 9. Find the numbers at which $f(x) = \begin{cases} x+1 & \text{if } x<0 \\ e^x & \text{if } 0 \le x \le 1 \\ 2-x & \text{if } x>1 \end{cases}$ is discontinuous.



or: f is continuous for all x except possibly at $x=0, 1$

$x=0$: $\lim_{x\to 0^-} f(x) = 1$, $\lim_{x\to 0^+} f(x) = 1$ $\therefore$ $\lim_{x\to 0} f(x) = 1 = f(0)$

$\therefore$ f is cont. at $x=0$

$x=1$ $\lim_{x\to 1^-} f(x) = e$, $\lim_{x\to 1^+} f(x) = 1$

$\therefore$ $\lim_{x\to 1} f(x)$ DNE

$\therefore$ f is discont. at $x=1$

NPC

$\boxed{x=1}$

$\boxed{6}$

(10) 10. Find the derivative of the function $f(x) = \sqrt{1-x}$ using the definition of the derivative $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$. (0 credit for using a formula for the derivative).

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{1-x-h}-\sqrt{1-x}}{h} \quad (4)$$

$$= \lim_{h\to 0} \frac{\sqrt{1-x-h}-\sqrt{1-x}}{h} \cdot \frac{\sqrt{1-x-h}+\sqrt{1-x}}{\sqrt{1-x-h}+\sqrt{1-x}}$$

−1pt for early omission of $\lim_{h\to 0}$

$$= \lim_{h\to 0} \frac{1-x-h-(1-x)}{h(\sqrt{1-x-h}+\sqrt{1-x})} = \lim_{h\to 0} \frac{-h}{h(\sqrt{1-x-h}+\sqrt{1-x})}$$

$$= \underbrace{\lim_{h\to 0} \frac{-1}{\sqrt{1-x-h}+\sqrt{1-x}}}_{(4)} = \underbrace{-\frac{1}{2\sqrt{1-x}}}_{(2)}$$

$$\boxed{-\frac{1}{2\sqrt{1-x}}}$$

$\boxed{10}$

(6) 11. Find an equation of the tangent line to the curve $y = x + \cos x$ at the point $(0, 1)$.

NPC

$$\frac{dy}{dx} = 1 - \sin x$$

$$\left.\frac{dy}{dx}\right|_{x=0} = 1 \qquad y - 1 = 1(x-0)$$

$$\boxed{y = x+1}$$

$\boxed{6}$

(16) 12. Find the derivatives of the following functions. Do not simplify.

4 pts each NPC

(a) $f(x) = 3e^x - \sqrt[3]{x^2}$.

$= 3e^x - x^{2/3}$

$$\boxed{f'(x) = 3e^x - \frac{2}{3}x^{-\frac{1}{3}}}$$

$\boxed{4}$

(b) $y = \frac{x \sin x}{e^x}$.

$$\boxed{\frac{dy}{dx} = \frac{e^x(x\cos x + \sin x) - e^x x \sin x}{e^{2x}}}$$

$\boxed{4}$

(c) $h(\theta) = \csc\theta + e^\theta \cot\theta$.

$$\boxed{h'(\theta) = -\csc\theta\cot\theta + e^\theta(-\csc^2\theta) + e^\theta\cot\theta}$$

$\boxed{4}$

(d) $f(t) = \sqrt{t} + t^3 \tan t$.

$= t^{\frac{1}{2}} + t^3 \tan t$

$$\boxed{f'(t) = \frac{1}{2}t^{-\frac{1}{2}} + t^3\sec^2 t + 3t^2\tan t}$$

$\boxed{4}$