NAME GRADING KEY

STUDENT ID

RECITATION INSTRUCTOR

RECITATION TIME

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DIRECTIONS

1. Write your name, 10–digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes, calculators or any electronic devices may be used on this exam.

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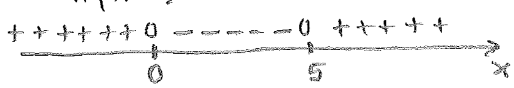
(5) 1. Find the domain of the function $h(x) = \dfrac{1}{\sqrt[4]{x^2 - 5x}}$.

Write your answer in the form of interval(s).

$x^2 - 5x > 0$ 2

$x(x-5) > 0$



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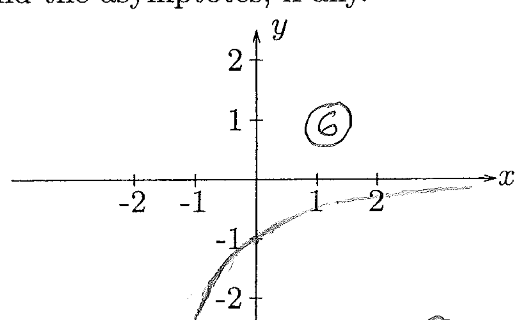
$\boxed{(-\infty, 0) \cup (5, \infty)}$

5

(11) 2. (a) Make a rough sketch of the graph of the function $y = f(x) = -e^{-x}$. Show clearly where the graph intersects the coordinate axes, and the asymptotes, if any.



(b) True or False. (Circle T or F)

1 pt each

(i) f is a one–to–one function. T F

(ii) f is an even function. T F

(iii) The range of f is $(-\infty, 0)$. T F

(iv) The domain of f^{-1} is $(0, \infty)$. T F

(v) f is increasing on $(-\infty, \infty)$. T F

11

(6) 3. If $f(x) = 2x^3 + 3$, find a formula for the inverse function f^{-1}.

$y = f(x) \iff x = f^{-1}(y)$

$y = 2x^3 + 3$

$x^3 = \frac{y-3}{2}$

$x = \sqrt[3]{\frac{y-3}{2}}$

$\therefore f^{-1}(y) = \sqrt[3]{\frac{y-3}{2}} \to f^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$

-3 pts for $f^{-1}(x) = \sqrt{\frac{y-3}{2}}$

$\boxed{f^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}}$ 6

6

(8) 4. Find the exact value of each expression

2pts each NPC

(a) $e^{2\ln 3} = e^{\ln 9} = 9$

$\boxed{9}$

(b) $\log_{10} 25 + \log_{10} 4 = \log_{10}(25 \cdot 4) = \log_{10} 100 = 2$

$\boxed{2}$

(c) $\sin \frac{5\pi}{4} = -\frac{1}{\sqrt{2}}$

$\boxed{-\frac{1}{\sqrt{2}}}$

(d) $\tan(-\pi e^{-\ln 4}) = \tan\left(-\frac{\pi}{e^{\ln 4}}\right) = \tan\left(-\frac{\pi}{4}\right) = -1$

$\boxed{-1}$

8

(6) 5. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation $2\cos x + \sin 2x = 0$.

$2\cos x + 2\sin x \cos x = 0$ 2

$\cos x\,(1 + \sin x) = 0$

$\cos x = 0 \longrightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

or $\sin x = -1 \longrightarrow x = \frac{3\pi}{2}$

2 2

$\boxed{\frac{\pi}{2}, \quad \frac{3\pi}{2}}$

6

(4) 6. If a ball is thrown straight up into the air with a velocity of 50 ft/sec, its height in feet after t seconds is given by $y = 50t - 16t^2$. Find the velocity when $t = 3$.

$v(t) = 50 - 32t$

$v(3) = 50 - 32 \cdot 3 = 50 - 96 = -46$ ft/sec

4 NPC

$\boxed{-46 \text{ ft/sec}}$

4

(7) 7. Circle the interval in which you are sure that the equation $x^4 + 4x - 25 = 0$ has a solution. State the name of the theorem you are using.

Let $f(x) = x^4 + 4x - 25$ $\quad$ f is continuous for all x

$f(0) = -25 < 0$ $\quad$ $f(3) = 81 + 12 - 25 = 68 > 0$

$f(1) = -20 < 0$ $\quad$ $f(4) = 256 + 16 - 25 = 247 > 0$

$f(2) = 16 + 8 - 25 = -1 < 0$

[0, 1]

[1, 2] 5

[2, 3]

[3, 4]

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Theorem: Intermediate Value Theorem 2

(10) 8. For each of the following, fill in the boxes below with a finite number, or one of the symbols $+\infty$, $-\infty$, or DNE (does not exist). It is not necessary to give reasons for your answers.

2 pts each NPC

(a) $\lim_{x\to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{4+x} = \lim_{x\to -4} \dfrac{\frac{x+4}{4x}}{x+4} = \lim_{x\to -4} \dfrac{1}{4x} = -\dfrac{1}{16}$

$\boxed{-\frac{1}{16}}$

(b) $\lim_{x\to 1} \dfrac{2-x}{(x-1)^2} = \infty$ ($2-x \to 1$, $(x-1)^2 \to 0^+$)

$\boxed{\infty}$

(c) $\lim_{h\to 0} \dfrac{(4+h)^2-16}{h} = \lim_{h\to 0} \dfrac{16+8h+h^2-16}{h}$
$= \lim_{h\to 0} \dfrac{h(8+h)}{h} = \lim_{h\to 0} (8+h) = 8$

$\boxed{8}$

(d) $\lim_{x\to -2} \dfrac{2-|x|}{2+x} = \lim_{x\to -2} \dfrac{2+x}{2+x} = 1$

x is near -2 $\therefore x<0$ $\therefore |x|=-x$

$\boxed{1}$

(e) $\lim_{x\to 0} \left(\dfrac{1}{x} - \dfrac{3}{x^2+3x}\right) = \lim_{x\to 0} \dfrac{x^2+3x-3x}{x(x^2+3x)} =$
$= \lim_{x\to 0} \dfrac{x^2}{x^2(x+3)} = \lim_{x\to 0} \dfrac{1}{x+3} = \dfrac{1}{3}$

$\boxed{\frac{1}{3}}$

$\boxed{10}$

(6) 9. Write the equations of the vertical and horizontal asymptotes, if any, of the graph of $y = \dfrac{x^2+1}{x^2-1}$.

$\lim_{x\to \pm\infty} \dfrac{x^2+1}{x^2-1} = \lim_{x\to\pm\infty} \dfrac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}} = 1$ $\therefore$ $y=1$ is a HA

$\lim_{x\to 1^+} \dfrac{x^2+1}{(x+1)(x-1)} = \infty$ $\therefore$ $x=1$ is a VA ($x^2+1 \to 2$, $x+1\to 2$, $x-1 \to 0^+$)

$\lim_{x\to (-1)^-} \dfrac{x^2+1}{(x+1)(x-1)} = \infty$ $\therefore$ $x=-1$ is a VA ($x^2+1 \to 2$, $x+1\to 0^-$, $x-1 \to -2$)

or graph is symmetric about the y-axis

Vertical asymptotes
$x=1$ (2) , $x=-1$ (2)
Horizontal asymptotes
$y=1$ (2)

$\boxed{6}$

(6) 10. Consider the function $f(x) = \begin{cases} \dfrac{x^2-x}{x^2-1} & \text{if } x \neq 1 \\ A & \text{if } x = 1 \end{cases}$, where A is a constant.

Find the value of A for which f is continuous at $x=1$.

f is continuous at 1 if $\lim_{x\to 1} f(x) = f(1)$

$\lim_{x\to 1} \dfrac{x^2-x}{x^2-1} = \lim_{x\to 1} \dfrac{x(x-1)}{(x+1)(x-1)} = \lim_{x\to 1} \dfrac{x}{x+1} = \dfrac{1}{2}$ (3)

$\therefore A = \frac{1}{2}$

0 pts for problem if no limit is indicated in work.

(3)

$\boxed{A = \frac{1}{2}}$

$\boxed{6}$

(10) 11. Find the derivative of the function $f(x) = x^3 + x$ using the definition of the derivative $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. (0 credit for using a formula for the derivative).

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 + (x+h) - (x^3+x)}{h} \quad (4)$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + x + h - x^3 - x}{h}$$

$$= \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2 + 1)}{h}$$

$$= \lim_{h \to 0} (3x^2 + 3xh + h^2 + 1) \quad (4)$$

$$= 3x^2 + 1 \quad (2)$$

$\boxed{3x^2+1}$ 10

(6) 12. Find the equation of the tangent line to the curve $y = 1 - x^3$ at the point $(0, 1)$.

$$\left.\begin{aligned} \frac{dy}{dx} &= -3x^2 \\ \left.\frac{dy}{dx}\right|_{x=0} &= 0 \end{aligned}\right\} (3)$$

equation of tangent line at $(0,1)$:

$$y - 1 = 0(x - 0)$$
$$y = 1$$

NPC

(3)

$\boxed{y=1}$ 6

(15) 13. Find the derivatives of the following functions. Do not simplify.

5 pts each NPC

(a) $y = \dfrac{x}{\sin x}$.

$$\boxed{\frac{dy}{dx} = \frac{\sin x - x\cos x}{\sin^2 x}}$$

(b) $f(x) = \sqrt{x}\tan x$.

$$\boxed{f'(x) = \sqrt{x}\sec^2 x + \frac{1}{2\sqrt{x}}\tan x}$$

(c) $h(\theta) = \dfrac{\sec\theta}{1 + \sec\theta}$.

$$\boxed{h'(\theta) = \frac{(1+\sec\theta)\sec\theta\tan\theta - \sec\theta\sec\theta\tan\theta}{(1+\sec\theta)^2}}$$

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