NAME GRADING KEY

STUDENT ID

RECITATION INSTRUCTOR

RECITATION TIME

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DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

(16) 1. Find the derivative of the following functions. (It is not necessary to simplify).

(a) $y = e^{-5x}\cos 3x$

NPC

$$\frac{dy}{dx} = e^{-5x}(-\sin 3x)3 + (-5)e^{-5x}\cos 3x$$

or → $\boxed{-3e^{-5x}\sin 3x - 5e^{-5x}\cos 3x}$ 4

(b) $F(x) = (x^3 + 4x)^7$

$\boxed{7(x^3+4x)^6(3x^2+4)}$ 4

(c) $f(x) = \sin^{-1}(\ln x)$

$$f'(x) = \frac{1}{\sqrt{1-(\ln x)^2}}\,\frac{1}{x}$$

or → $\boxed{\dfrac{1}{x\sqrt{1-(\ln x)^2}}}$ 4

(d) $y = \sqrt{1 + \sin^2(3x)}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1+\sin^2(3x)}}\,2\sin(3x)\cos(3x)\cdot 3$$

or → $\boxed{\dfrac{3\sin(3x)\cos(3x)}{\sqrt{1+\sin^2(3x)}}}$ 4

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(8) 2. Find $\frac{dy}{dx}$ by implicit differention, if $xe^y = y - 1$.

$$xe^y \frac{dy}{dx} + e^y = \frac{dy}{dx} \quad ④$$

$$(xe^y - 1)\frac{dy}{dx} = -e^y$$

$$\frac{dy}{dx} = -\frac{e^y}{xe^y - 1} \quad ④ \text{ or } \quad \boxed{\frac{dy}{dx} = \frac{e^y}{1 - xe^y}} \quad \boxed{8}$$

(9) 3. Find an equation of the tangent line to the ellipse $\frac{x^2}{9} + \frac{y^2}{36} = 1$ at the point $(-1, 4\sqrt{2})$.

$$\frac{2x}{9} + \frac{2y}{36}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{4x}{y}$$

At $(-1, 4\sqrt{2})$: $\frac{dy}{dx} = -\frac{4(-1)}{4\sqrt{2}} = \frac{1}{\sqrt{2}}$ ⑥

$$y - 4\sqrt{2} = \frac{1}{\sqrt{2}}(x+1) \quad ③ \qquad \boxed{y - 4\sqrt{2} = \frac{1}{\sqrt{2}}(x+1)} \quad \boxed{9}$$

(9) 4. Evaluate each expression:

NPC

(a) $\cos^{-1}\left(\frac{1}{2}\right) = y \iff \cos y = \frac{1}{2},\ 0 \le y \le \pi$ $\quad \boxed{\frac{\pi}{3}}$ ③

(b) $\tan^{-1}(-1) = y \iff \tan y = -1,\ -\frac{\pi}{2} < y < \frac{\pi}{2}$ $\quad \boxed{-\frac{\pi}{4}}$ ③

(c) $\tan\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)$ $\quad y = \sin^{-1}\frac{\sqrt{3}}{2} \iff \sin y = \frac{\sqrt{3}}{2},\ -\frac{\pi}{2} \le y \le \frac{\pi}{2}$ $\quad \boxed{\sqrt{3}}$ ③

$\therefore y = \frac{\pi}{3}$

$\tan\frac{\pi}{3} = \sqrt{3}$

(6) 5. Find the derivative of $y = x^x$.

$$y = x^x = e^{x\ln x} \quad ②$$

$$\frac{dy}{dx} = e^{x\ln x}\left(x\frac{1}{x} + \ln x\right)$$

$$= e^{x\ln x}(1 + \ln x)$$

$$= x^x(1 + \ln x)$$

④

or $\ln y = x\ln x$ ②

$$\frac{1}{y}\frac{dy}{dx} = x\frac{1}{x} + \ln x$$

$$\frac{dy}{dx} = y(1 + \ln x)$$

$$= x^x(1 + \ln x) \quad ④$$

or $\boxed{x^x(1 + \ln x)}$ $\boxed{6}$

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(6) 6. Find the second derivative of $h(x) = \tan^{-1}(x^2)$.

$$h'(x) = \frac{1}{1+x^4}\,2x = \frac{2x}{1+x^4} \quad 3$$

$$h''(x) = \frac{(1+x^4)2 - 2x\,4x^3}{(1+x^4)^2}$$

$$= \frac{-6x^4+2}{(1+x^4)^2}$$

or 3

$$\boxed{\frac{-6x^4+2}{(1+x^4)^2}}$$

$\boxed{6}$

(10) 7. The position of a particle is given by the equation
$s = 5\cos 2t$.
Find all values of t in the interval $[0, \pi]$ for which

-1pt for each additional wrong value

(a) the velocity is 0.

$$v = \frac{ds}{dt} = -10\sin 2t \quad 3$$

$v = 0$: $\sin 2t = 0 \rightarrow 2t = 0, \pi, 2\pi$

$t = 0, \frac{\pi}{2}, \pi$

1 1 1

$$\boxed{0, \ \frac{\pi}{2}, \ \pi}$$

(b) the acceleration is 0.

$$a = \frac{dv}{dt} = -20\cos 2t \quad 2$$

$a = 0$: $\cos 2t = 0 \quad 2t = \frac{\pi}{2}, \frac{3\pi}{2}$

$t = \frac{\pi}{4}, \frac{3\pi}{4}$

1 1

$$\boxed{\frac{\pi}{4}, \ \frac{3\pi}{4}}$$

$\boxed{10}$

(10) 8. Gravel is being dumped from a conveyor belt at the rate of 30 ft^3/min and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high? ($V = \frac{1}{3}\pi r^2 h$).

D = base diameter, h = height, $D = h$, $\frac{dV}{dt} = 30\ ft^3/min$ 1

Find $\frac{dh}{dt}$ when $h = 10$.

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{D}{2}\right)^2 h = \frac{\pi}{12}h^3 \quad 4$$

$$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt} \quad 2$$

When $h = 10$:

$$30 = \frac{\pi}{4}(10)^2\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{120}{100\pi} = \frac{6}{5\pi}\ ft/min \quad 3$$

$$\boxed{\frac{6}{5\pi}\ ft/min}$$

$\boxed{10}$

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(12) 9. A snowball melts so that its surface area decreases at the rate of 2 cm^2/min. How fast is the volume decreasing when the radius is 8 cm? ($V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$).

V = volume, S = surface area, r = radius

$\frac{dS}{dt} = -2\ cm^2/min$ ① Find $\frac{dV}{dt}$ when $r = 8cm$

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$ ② $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ ②

$\frac{dr}{dt} = \frac{1}{8\pi r}\frac{dS}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{1}{8\pi r}\frac{dS}{dt}$

$\frac{dV}{dt} = \frac{r}{2}\frac{dS}{dt}$ ③

When $r=8$: $\frac{dV}{dt} = \frac{8}{2}(-2) = -8$ ③

or

When $r = 8$:

$-2 = 8\pi 8 \frac{dr}{dt} \rightarrow \frac{dr}{dt} = -\frac{1}{32\pi}$ ③

$\frac{dV}{dt} = 4\pi 8^2\left(-\frac{1}{32\pi}\right) = -8$ ③

The volume is decreasing at the rate of ① $\boxed{8\ cm^3/min}$ $\boxed{12}$

(8) 10. Use a differential (or equivalently a linear approximation) to estimate $\sqrt{36.1}$.

$f(x) \approx f(a) + f'(a)(x-a)$ ②

Let $f(x) = \sqrt{x}$, $a = 36$. $f'(x) = \frac{1}{2\sqrt{x}}$

$\sqrt{x} \approx 6 + \frac{1}{2\cdot 6}(x-36)$ ③

With $x=36.1$: $\sqrt{36.1} \approx 6 + \frac{1}{12}(36.1-36) =$

$= 6 + \frac{1}{120}$ or $= \frac{721}{120}$ ③

or $f(x+dx) \approx f(x) + dy$ ② where $dy = f'(x)dx$

Let $f(x) = \sqrt{x}$: $\sqrt{x+dx} \approx \sqrt{x} + \frac{1}{2\sqrt{x}}dx$ ③

With $x = 36$ and $dx = 0.1$:

$\sqrt{36.1} \approx \sqrt{36} + \frac{1}{2\sqrt{36}}(0.1) = 6 + \frac{1}{120}$ or ③

$\boxed{6 + \frac{1}{120} \text{ or } \frac{721}{120}}$ $\boxed{8}$

(6) 11. Find the differential dy if

(a) $y = \tan(3x)$

$dy = \sec^2(3x)\cdot 3\,dx$

−1 pt for each missing dx

$\boxed{dy = 3\sec^2(3x)\,dx}$ ③

(b) $y = x\sec^2 x$

$dy = [x\, 2(\sec x)(\sec x)\tan x + \sec^2 x]\,dx$

or

$\boxed{dy = \sec^2 x\,(2x\tan x + 1)\,dx}$ ③