NAME GRADING KEY

STUDENT ID ______

RECITATION INSTRUCTOR ______

RECITATION TIME ______

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DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

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(16) 1. Find the derivatives of the following functions. It is not necessary to simplify.

(a) $y = e^{-5x}\cos 3x$

$$\frac{dy}{dx} = e^{-5x}(-\sin 3x)\cdot 3 + (-5)e^{-5x}\cos 3x$$

or

$$\boxed{-3e^{-5x}\sin 3x - 5e^{-5x}\cos 3x}$$

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(b) $f(x) = \sin^{-1}[(\ln x)^2]$

$$f'(x) = \frac{1}{\sqrt{1-(\ln x)^4}}\, 2\ln x \cdot \frac{1}{x}$$

or

$$\boxed{\frac{2\ln x}{x\sqrt{1-(\ln x)^4}}}$$

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(c) $y = \ln(\sec x + \tan x)$

$$\frac{dy}{dx} = \frac{1}{\sec x + \tan x}(\sec x \tan x + \sec^2 x)$$

or

$$\boxed{\sec x}$$

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(d) $F(x) = \dfrac{\sin^2 x}{1+x^2}$

$$F'(x) = \frac{(1+x^2)2\sin x\cos x - 2x\sin^2 x}{(1+x^2)^2}$$

or

$$\boxed{\frac{2(1+x^2)\sin x\cos x - 2x\sin^2 x}{(1+x^2)^2}}$$

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NPC but −1 pt if first answer is correct and there is an error in copying or simplifying

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(10) 2. Find an equation of the tangent line to each curve at the given point.

(a) $y = \sin(\sin x)$ at the point $(\pi, 0)$.

$$\frac{dy}{dx} = \cos(\sin x) \cdot \cos x \quad 2$$

$$\left.\frac{dy}{dx}\right|_{x=\pi} = \cos(\sin\pi)\cos\pi = \cos(0)\cos\pi = -1 \quad 1$$

$$y - 0 = -1(x - \pi) \quad 2 \quad \text{or} \quad \boxed{y = -x + \pi} \quad \boxed{5}$$

(b) $y = \ln(\ln x)$ at the point $(e, 0)$.

$$\frac{dy}{dx} = \frac{1}{\ln x}\,\frac{1}{x} \quad 2$$

$$\left.\frac{dy}{dx}\right|_{x=e} = \frac{1}{\ln e}\,\frac{1}{e} = \frac{1}{e} \quad 1$$

$$y - 0 = \frac{1}{e}(x - e) \quad \text{or} \quad 2 \quad \boxed{y = \frac{1}{e}x - 1} \quad \boxed{5}$$

(9) 3. Find the exact value of each expression.

(a) $\tan^{-1}(-\sqrt{3}) = y \iff \tan y = -\sqrt{3}, \; -\frac{\pi}{2} < y < \frac{\pi}{2}$ NPC

$$y = -\frac{\pi}{3} \qquad \boxed{-\frac{\pi}{3}} \quad 3$$

(b) $\sin^{-1}(\sin(\frac{4\pi}{3})) = \sin^{-1}(-\frac{\sqrt{3}}{2}) = y$

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = y \iff \sin y = -\frac{\sqrt{3}}{2}, \; -\frac{\pi}{2} \le y \le \frac{\pi}{2} \qquad \boxed{-\frac{\pi}{3}} \quad 3$$

$$y = -\frac{\pi}{3}$$

(c) $\cos(\cos^{-1}(0.2)) = \cos y$

$$y = \cos^{-1}(0.2) \iff \cos y = 0.2, \; 0 \le y \le \pi \qquad \boxed{0.2} \quad 3$$

(4) 4. Find the differential dy of the function $y = x \ln x$.

NPC

$$dy = \frac{dy}{dx}dx = \left(x\frac{1}{x} + \ln x\right)dx \quad \text{or} \quad \boxed{dy = (1 + \ln x)dx} \quad \boxed{4}$$

(6) 5. Find the second derivative of the function $h(x) = \tan^{-1}(x^2)$.

$$h'(x) = \frac{1}{1 + x^4}\,2x = \frac{2x}{1 + x^4} \quad 2$$

$$h''(x) = \frac{(1 + x^4)2 - 2x\,4x^3}{(1 + x^4)^2}$$

$$\text{or} \quad 4 \quad \boxed{h''(x) = \frac{2 - 6x^4}{(1 + x^4)^2}} \quad \boxed{6}$$

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(6) 6. If $\sin(y^2) = xy$, find $\frac{dy}{dx}$ using implicit differentiation.

$$\cos(y^2)\cdot 2y\frac{dy}{dx} = x\frac{dy}{dx} + y \quad (2)\ (2)$$

$$\left(2y\cos(y^2) - x\right)\frac{dy}{dx} = y$$

$$\frac{dy}{dx} = \boxed{\frac{y}{2y\cos(y^2) - x}} \quad (2)$$

6

(10) 7. Find an equation of the tangent line to the curve $x^2y^2 = (y+1)^2(4-y^2)$ at the point $(0,-2)$.

$$x^2y^2 = (y+1)^2(4-y^2)$$

$$x^2 2y\frac{dy}{dx} + 2xy^2 = (y+1)^2(-2y)\frac{dy}{dx} + (4-y^2)2(y+1)\frac{dy}{dx} \quad (2)\ (2)\ (2)$$

When $(x,y) = (0,-2)$:

$$0 + 0 = (-2+1)^2(-2)(-2)\frac{dy}{dx} + (4-4)2(-2+1)\frac{dy}{dx}$$

$$\frac{dy}{dx} = 0 \quad (2)$$

$$y - (-2) = 0(x-0) \quad (2) \text{ or } \boxed{y = -2}$$

10

(12) 8. (a) Find the linear approximation of $f(x) = \sec x$ at $a = \frac{\pi}{4}$.

$$f(x) \approx f\left(\tfrac{\pi}{4}\right) + f'\left(\tfrac{\pi}{4}\right)\left(x - \tfrac{\pi}{4}\right), \text{ for } x \text{ near } \tfrac{\pi}{4} \quad (4)$$

$$f\left(\tfrac{\pi}{4}\right) = \sec\tfrac{\pi}{4} = \sqrt{2}$$

$$f'(x) = \sec x \tan x$$

$$f'\left(\tfrac{\pi}{4}\right) = \sec\tfrac{\pi}{4}\cdot\tan\tfrac{\pi}{4} = \sqrt{2}$$

(2) (2)

$$\boxed{\sec x \approx \sqrt{2} + \sqrt{2}\left(x - \tfrac{\pi}{4}\right), \text{ for } x \text{ near } \tfrac{\pi}{4}}$$

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(b) Estimate $\sec 47°$.

$$47° = 47\cdot\frac{\pi}{180} \text{ rads}$$

$$\sec\frac{47\pi}{180} \approx \sqrt{2} + \sqrt{2}\left(\frac{47\pi}{180} - \frac{\pi}{4}\right) = \sqrt{2} + \sqrt{2}\,\frac{2\pi}{180}$$

or or

$$\boxed{\sec 47° \approx \sqrt{2} + \frac{\sqrt{2}\pi}{90}}$$

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NPC but full credit if error is due to minor numerical error in answer to part (a)

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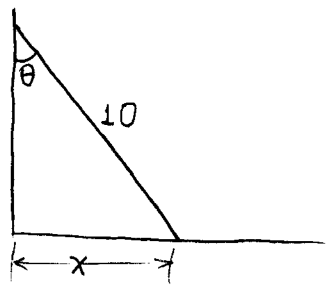
(12) 9. A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2 ft/sec, how fast is the angle between the top of the ladder and the wall changing when the angle is $\pi/4$ rad?



$\frac{dx}{dt} = 2$ ft/sec ②

Find $\frac{d\theta}{dt}$ when $\theta = \frac{\pi}{4}$

$\sin\theta = \frac{x}{10}$ ③

$\cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt}$ ④

When $\theta = \frac{\pi}{4}$: $\frac{1}{\sqrt{2}}\frac{d\theta}{dt} = \frac{1}{10}\cdot 2$

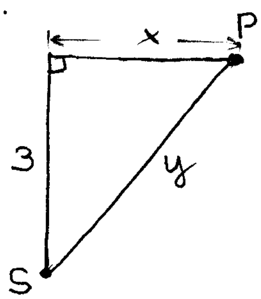
$\frac{d\theta}{dt} = \frac{\sqrt{2}}{5}$

③

$\boxed{\frac{\sqrt{2}}{5} \text{ rad/sec}}$

$\boxed{12}$

(15) 10. A plane flying horizontally at an altitude of 3 mi passes directly over a radar station. The distance between the station and the plane is increasing at a rate of 600 mi/hr. Find the speed of the plane when the distance between the plane and the station is 5 mi.



$\frac{dy}{dt} = 600$ mi/hr ②

Find $\frac{dx}{dt}$ when $y = 5$ mi

$x^2 + 9 = y^2$ ③

$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$ ④

When $y = 5$: $x = \sqrt{25-9} = 4$ ③

$4\frac{dx}{dt} = 5 \cdot 600$

$\frac{dx}{dt} = 750$

③

$\boxed{750 \text{ mi/hr}}$

$\boxed{15}$