NAME GRADING KEY

10-DIGIT PUID ______________

RECITATION INSTRUCTOR ______________

RECITATION TIME ______________

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TOTAL	/100

DIRECTIONS

1. Write your name, 10-digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

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(16) 1. Find the derivative of the following functions. (It is not necessary to simplify). NPC

(a) $y = e^{-5x}\cos(3x)$.

$$\frac{dy}{dx} = e^{-5x}(-\sin(3x))3 + \cos(3x)e^{-5x}(-5)$$

or → $\boxed{-3e^{-5x}\sin(3x) - 5e^{-5x}\cos(3x)}$ 4

(b) $y = e^{e^x}$

$$\frac{dy}{dx} = e^{e^x}e^x$$

$\boxed{e^{e^x}e^x}$ 4

(c) $y = \ln(1 + 2e^{3x})$.

$$\frac{dy}{dx} = \frac{1}{1+2e^{3x}}\,2e^{3x}\cdot 3$$

$\boxed{\dfrac{6e^{3x}}{1+2e^{3x}}}$ 4

(d) $f(x) = \sqrt[3]{9 + 8\sin 2x}$ $=(9+8\sin 2x)^{1/3}$

$$f'(x) = \frac{1}{3}(9+8\sin 2x)^{-\frac{2}{3}}\cdot 8(\cos 2x)2$$

$\boxed{\dfrac{16}{3}(9+8\sin 2x)^{-\frac{2}{3}}\cos 2x}$ 4

(8) 2. Find $\frac{dy}{dx}$ by implicit differentiation, if $(\tan y)(\sin x) = xy$.

$$\underbrace{(\tan y)(\cos x) + (\sin x)(\sec^2 y)\frac{dy}{dx}}_{3} = \underbrace{x\frac{dy}{dx} + y}_{2}$$

$$[(\sin x)\sec^2 y - x]\frac{dy}{dx} = y - (\tan y)(\cos x)$$

3 $\boxed{\frac{dy}{dx} = \frac{y-(\tan y)(\cos x)}{(\sin x)\sec^2 y - x}}$ $\boxed{8}$

(12) 3. Find the exact value of each expression.

(a) $\sin^{-1}(\frac{\sqrt{3}}{2}) = y \iff \sin y = \frac{\sqrt{3}}{2}, -\frac{\pi}{2} \le y \le \frac{\pi}{2}$ NPC

$y = \frac{\pi}{3}$

$\boxed{\frac{\pi}{3}}$ 3

(b) $\sin(\sin^{-1} 0.7) = 0.7$

$\boxed{0.7}$ 3

(c) $\tan^{-1}(\tan \frac{4\pi}{3}) = \tan^{-1}(\sqrt{3}) = y \iff \tan y = \sqrt{3}, -\frac{\pi}{2} < y < \frac{\pi}{2}$

$y = \frac{\pi}{3}$

$\boxed{\frac{\pi}{3}}$ 3

(d) $\cos^{-1}(-\frac{1}{2}) = y \iff \cos y = -\frac{1}{2}, 0 \le y \le \pi$

$y = \frac{2\pi}{3}$

$\boxed{\frac{2\pi}{3}}$ 3

(12) 4. Find the derivatives of the following functions. (It is not necessary to simplify). NPC

(a) $y = \tan^{-1}\sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{(\sqrt{x})^2+1} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{(x+1)2\sqrt{x}}$$

or→ $\boxed{\frac{1}{2\sqrt{x}(x+1)}}$ 4

(b) $f(x) = \sin^{-1}(x^2)$

$$f'(x) = \frac{1}{\sqrt{1-x^4}} 2x$$

$\boxed{\frac{2x}{\sqrt{1-x^4}}}$ 4

(c) $y = (x+1)^x = e^{\ln(x+1)^x} = e^{x\ln(x+1)}$

$$\frac{dy}{dx} = e^{x\ln(x+1)}\left(x\frac{1}{x+1} + \ln(x+1)\right)$$

or→ $\boxed{(x+1)^x\left(\frac{x}{x+1} + \ln(x+1)\right)}$ 4

(8) 5. Find a formula for $f^{(n)}(x)$ if $f(x) = \frac{1}{x-1}$.

$f(x) = (x-1)^{-1}$
$f^{(1)}(x) = -1(x-1)^{-2}$
$f^{(2)}(x) = 1\cdot 2(x-1)^{-3}$
$f^{(3)}(x) = -1\cdot 2\cdot 3(x-1)^{-4}$
$f^{(4)}(x) = 1\cdot 2\cdot 3\cdot 4(x-1)^{-5}$
. . . .

$f^{(n)}(x) = (-1)^n n!\,(x-1)^{-(n+1)}$

$\boxed{f^{(n)}(x) = \underbrace{(-1)^n}_{2}\ \underbrace{n!}_{3}\ \underbrace{(x-1)^{-(n+1)}}_{3}}$

8

(4) 6. Find an equation of the tangent line to the curve $y = \sinh x$ at the point $(0,0)$.

$y = \sinh x$

NPC

$\frac{dy}{dx} = \cosh x$

$\left.\frac{dy}{dx}\right|_{x=0} = 1$

$\boxed{y = x}$

4

(6) 7. If $F(x) = f(g(x))$, $f'(1) = 5$, and $g(x) = e^{2x}$, find $F'(0)$.

$F'(x) = f'(g(x))\,g'(x)$ 2

$= f'(e^{2x})\,2e^{2x}$

$F'(0) = f'(1)\cdot 2\cdot 1 = 5\cdot 2\cdot 1 = 10$
4

$\boxed{F'(0) = 10}$

6

(6) 8. Find the linearization $L(x)$ of the function $f(x) = (\sin x + \cos x)^3$ at $a = \frac{\pi}{2}$.

$L(x) = f(a) + f'(a)(x-a)$
$= f(\frac{\pi}{2}) + f'(\frac{\pi}{2})(x - \frac{\pi}{2})$ or 2

$f(x) = (\sin x + \cos x)^3,\ f(\frac{\pi}{2}) = 1$

$f'(x) = 3(\sin x + \cos x)^2(\cos x - \sin x)$

$f'(\frac{\pi}{2}) = 3\cdot 1\cdot(-1) = -3$

$\boxed{L(x) = \overbrace{1}^{1}\ \overbrace{-3\left(x - \frac{\pi}{2}\right)}^{3}}$

6

(4) 9. Find the differential dy if $y = \sec(5x)$.

$dy = \sec(5x)\tan(5x)\cdot 5\,dx$

$\boxed{dy = \overbrace{5\sec(5x)\tan(5x)}^{3}\ \overbrace{dx}^{1}}$

4

(12) 10. Air is let out of a spherical balloon so that its surface area is decreasing at a rate of 2 cm^2/sec. Find the rate at which the radius of the balloon is decreasing when the radius is 20 cm.

Let S be the surface area and r be the radius of the balloon.

Given: $\frac{dS}{dt} = -2 \text{ cm}^2/\text{sec}$ (2)

Find $\frac{dr}{dt}$ when $r = 20\text{cm}$.

$S = 4\pi r^2$ (3)

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$ (3)

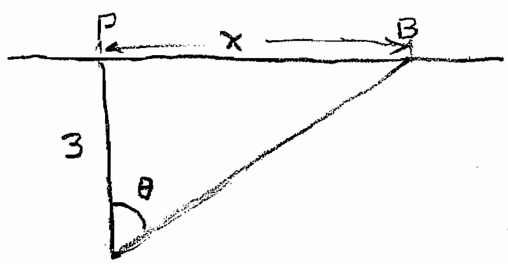
When $r = 20$: $-2 = 8\pi \, 20 \frac{dr}{dt}$

$\frac{dr}{dt} = -\frac{1}{80\pi}$ cm/sec (4)

$\boxed{\frac{1}{80\pi} \text{ cm/sec}}$

12

(12) 11. A lighthouse is located on a small island 3 km away from the nearest point P on a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P?



Given $\frac{d\theta}{dt} = 4 \cdot 2\pi = 8\pi$ rads/min (2)

Find $\frac{dx}{dt}$ when $x = 1$ km

$\tan\theta = \frac{x}{3}$ (3)

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{3}\frac{dx}{dt}$ (3)

When $x = 1$: $\cos\theta = \frac{3}{\sqrt{10}} \longrightarrow \sec\theta = \frac{\sqrt{10}}{3}$ (2)

$\left(\frac{\sqrt{10}}{3}\right)^2 \cdot 8\pi = \frac{1}{3}\frac{dx}{dt}$

$\frac{dx}{dt} = \frac{80\pi}{3}$ km/min (2)

$\boxed{\frac{80\pi}{3} \text{ km/min}}$

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