1. Find the derivative of the following functions. (It is not necessary to simplify).

   (a) \( y = (1 + \cos^2 x)^6 \)

   \[
   \frac{dy}{dx} = 6(1 + \cos^2 x)^5 2\cos x (-\sin x) \quad \Rightarrow \quad -12(1 + \cos^2 x)^5 \cos x \sin x
   \]

   (b) \( g(t) = \frac{1}{(t^4 + 1)^3} = (t^4 + 1)^{-3} \)

   \[
   g'(t) = -3(t^4 + 1)^{-4} 4t^3 \quad \Rightarrow \quad -12(t^4 + 1)^{-4} t^3
   \]

   (c) \( y = \sin^2(3x) \)

   \[
   \frac{dy}{dx} = 2 \sin(3x) \cos(3x) 3 \quad \Rightarrow \quad 6 \sin(3x) \cos(3x)
   \]

   (d) \( y = \ln(x^4 \tan x) \)

   \[
   \frac{dy}{dx} = \frac{1}{x^4 \tan x} (x^4 \sec^2 x + 4x^3 \tan x) \quad \Rightarrow \quad \frac{x^4 \sec^2 x + 4x^3 \tan x}{x^4 \tan x}
   \]

   \[
   \Rightarrow \quad y = 4 \ln x + \ln(\tan x)
   \]

   \[
   \frac{dy}{dx} = \frac{4}{x} + \frac{1}{\tan x} \sec^2 x
   \]
(8) 2. Find \( \frac{dy}{dx} \) by implicit differentiation, if \( x^2 y + xy^2 = 3x \).

\[
\frac{d}{dx} \left( x^2 y + xy^2 \right) = \frac{d}{dx} 3x
\]

\[
x^2 \frac{dy}{dx} + 2xy + y^2 + x \frac{dy}{dx} + 2xy \frac{dy}{dx} = 3
\]

\[
\frac{dy}{dx} = \frac{3 - 2xy - y^2}{x^2 + 2xy}
\]

(8) 3. Find the second derivative of \( y = xe^{2x} \).

\[
\frac{dy}{dx} = xe^{2x} + e^{2x}
\]

\[
\frac{d^2 y}{dx^2} = xe^{2x} + 2e^{2x} + 2e^{2x}
\]

(12) 4. Find the exact value of each expression.

(a) \( \sin^{-1} \left( -\frac{1}{\sqrt{2}} \right) = y \iff \sin y = -\frac{1}{\sqrt{2}} \), \( -\frac{\pi}{4} \leq y \leq \frac{\pi}{4} \).

\[
y = -\frac{\pi}{4}
\]

(b) \( \sec(\tan^{-1} 2) = \sec y \)

Let \( y = \tan^{-1} 2 \iff \tan y = 2 \), \( -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \).

\[
\tan^2 y + 1 = \sec^2 y \Rightarrow \sec y = \frac{\sqrt{5}}{2}, -\frac{\pi}{2} < y < \frac{\pi}{2}
\]

Because

\[
y = -\frac{\pi}{4}
\]

(c) \( \tan^{-1} \left( \tan \frac{5\pi}{6} \right) = y \iff \tan y = \tan \frac{5\pi}{6}, -\frac{\pi}{2} < y < \frac{\pi}{2} \)

\[
y = -\frac{\pi}{6}
\]

(d) \( \cos^{-1} 0 = y \iff \cos y = 0 \), \( 0 \leq y \leq \pi \)

\[
y = \frac{\pi}{2}
\]

(6) 5. Find the slope of the tangent line to the curve \( y = \sinh x \) at the point \( \left( \ln 2, \frac{3}{4} \right) \) and express your answer as the ratio of two integers.

\[
\frac{dy}{dx} = \cosh x = \frac{e^x + e^{-x}}{2}
\]

\[
\left. \frac{dy}{dx} \right|_{x = \ln 2} = \frac{e^{\ln 2} + e^{-\ln 2}}{2} = \frac{2 + \frac{1}{2}}{2} = \frac{5}{4}
\]

\[
\frac{5}{4}
\]
6. Find the derivatives of the following functions. (It is not necessary to simplify).

(a) \( y = \tan^{-1}(x^2) \)
\[
\frac{dy}{dx} = \frac{1}{(x^2)^2 + 1} \cdot 2x
\]

(b) \( f(x) = \sin^{-1}(2x + 1) \)
\[
f'(x) = \frac{1}{\sqrt{1 - (2x + 1)^2}} \cdot 2
\]

(c) \( y = x^{\sqrt{x}} \)
\[
\frac{dy}{dx} = e^{\ln x^{\sqrt{x}}} \cdot \left( \frac{\ln x^{\sqrt{x}}}{\sqrt{x}} + \frac{1}{2\sqrt{x}} \cdot \frac{1}{x} \right)
\]

7. Find the linearization \( L(x) \) of the function \( f(x) = x^5 \) at \( a = 1 \).
\[
L(x) = f(a) + f'(a) (x-a)
\]
\[
f(1) = 1, \quad f'(1) = 5
\]

8. Find the differential \( dy \) if

(a) \( y = \sqrt{1 + x^2} \)
\[
dy = \frac{1}{2\sqrt{1+x^2}} \cdot 2x \, dx
\]

(b) \( y = \sec(3x) \)
\[
dy = \sec(3x) \tan(3x) \cdot 3 \, dx
\]
9. A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/sec, how fast is the boat approaching the dock when it is 8 m from the dock.

Let \( x \) be the distance of the boat from the dock
and \( y \) be the length of the rope.

Given \( \frac{dx}{dt} = -1 \text{ m/sec} \) \( \tag{2} \)

Find \( \frac{dx}{dt} \) when \( x = 8 \text{ m} \)

\( x^2 + 1 = y^2 \) \( \tag{3} \)

\[ 2x \frac{dx}{dt} = 2y \frac{dy}{dt} \] \( \tag{3} \)

When \( x = 8 \): \( y = \sqrt{65} \)

\[ 2 \cdot 8 \frac{dx}{dt} = 2 \cdot \sqrt{65} \cdot (-1) \rightarrow \frac{dx}{dt} = -\frac{\sqrt{65}}{8} \text{ m/sec} \] \( \tag{4} \)

The boat is approaching the dock at the rate of \( \frac{\sqrt{65}}{8} \) m/sec \( \tag{12} \)

10. A coffee cup has the shape of an inverted circular cone with height 10 cm and radius at the top 5 cm. If coffee is poured into the cup at the rate of 2 cm\(^3\)/sec, how fast is the coffee level rising when the coffee is 5 cm deep?

Let \( V \) be the volume of the coffee in the cup and
\( h \) be the depth of the coffee.

Given \( \frac{dV}{dt} = 2 \text{ cm}^3/\text{sec} \) \( \tag{2} \)

Find \( \frac{dh}{dt} \) when \( h = 5 \text{ cm} \)

\[ V = \frac{1}{3} \pi r^2 h \] \( \tag{2} \)

\( \frac{r}{h} = \frac{5}{10} \rightarrow r = \frac{1}{2} h \) \( \tag{2} \)

\[ V = \frac{\pi}{12} h^3 \] \( \tag{3} \)

\[ \frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt} \]

When \( h = 5 \): \[ 2 = \frac{\pi}{4} \cdot 5^2 \frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{8}{25\pi} \text{ cm/sec} \] \( \tag{4} \)

The coffee level is rising at the rate of \( \frac{8}{25\pi} \) cm/sec \( \tag{12} \)