NAME GRADING KEY

STUDENT ID ______

RECITATION INSTRUCTOR ______

RECITATION TIME ______

| Page | Score |
|---|---|
| Page 1 | /16 |
| Page 2 | /30 |
| Page 3 | /26 |
| Page 4 | /28 |
| TOTAL | /100 |

DIRECTIONS

1. Write your name, 10–digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes, calculators or any electronic devices may be used on this exam.

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(16) 1. Find the derivatives of the following functions. (It is not necessary to simplify). NPC

(a) $y = e^{3x}\cos(4x)$

$\boxed{e^{3x}(-\sin(4x))\cdot 4 + 3e^{3x}\cos(4x)}$ (4)

(b) $f(t) = \sqrt[3]{1-\sin t}$

$= (1-\sin t)^{1/3}$

$\boxed{\frac{1}{3}(1-\sin t)^{-2/3}(-\cos t)}$ (4)

(c) $y = \tan(e^{3\theta})$

$\boxed{\sec^2(e^{3\theta})\cdot e^{3\theta}\cdot 3}$ (4)

(d) $y = \ln(\sin(e^x))$

$\boxed{\frac{1}{\sin(e^x)}\cos(e^x)\cdot e^x}$ (4)

(6) 2. If $F(x) = f(g(x))$, find $F'(1)$ if $f(1) = 3,\ g(1) = 2,\ f'(1) = 5,\ g'(1) = 6,$ $f'(2) = 4,\ g'(2) = 7$.

$F'(x) = f'(g(x))\, g'(x)$ ②

$F'(1) = f'(g(1))\, g'(1) = f'(2)\, g'(1)$

$= 4 \cdot 6 = 24$ ④

$\boxed{24}$ $\boxed{6}$

(9) 3. Find an equation of the tangent line to the curve $x^2 + y^2 = 3y + 8$ at the point $(-2, 4)$.

$2x + 2y\frac{dy}{dx} = 3\frac{dy}{dx}$ ④ $\quad \frac{dy}{dx} = \frac{2x}{3-2y}$

$\left.\frac{dy}{dx}\right|_{(-2,4)} = \frac{2(-2)}{3-8} = \frac{4}{5}$ ③

eq. of tan. line : $y - 4 = \frac{4}{5}(x+2)$ ②

$\boxed{y - 4 = \frac{4}{5}(x+2)}$ $\boxed{9}$

(9) 4. Find the exact value of

NPC

(a) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = y \iff \cos y = -\frac{\sqrt{3}}{2},\ 0 \le y \le \pi$

$y = \frac{5\pi}{6}$

$\boxed{\frac{5\pi}{6}}$ ③

(b) $\tan^{-1}(-\sqrt{3}) = y \iff \tan y = -\sqrt{3},\ -\frac{\pi}{2} < y < \frac{\pi}{2}$

$y = -\frac{\pi}{3}$

$\boxed{-\frac{\pi}{3}}$ ③

(c) $\sin(2\sin^{-1}(\frac{1}{2})) = \sin 2y = 2\sin y\cos y = 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

Let $y = \sin^{-1}(\frac{1}{2}) \iff \sin y = \frac{1}{2}\ ,\ -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\cos^2 y = 1 - \sin^2 y = 1 - (\frac{1}{2})^2 = \frac{3}{4}$

$\therefore \cos y = \cancel{\pm}\, \frac{\sqrt{3}}{2}$ $+$

$\boxed{\frac{\sqrt{3}}{2}}$ ③

(6) 5. Find the differential $dy$ of each function:

(a) $y = x\sec(3x)$

$dy = [x\sec(3x)\tan(3x)\cdot 3 + \sec(3x)]\,dx$

−1 pt for missing $dx$

$\boxed{dy = [x\sec(3x)\tan(3x)\cdot 3 + \sec(3x)]\,dx}$ ③

(b) $y = e^{\sqrt{t^2+1}}$

−1 pt for missing $dt$

$\boxed{dy = e^{\sqrt{t^2+1}} \frac{1}{2\sqrt{t^2+1}}\, 2t\, dt}$ ③

(12) 6. Find the derivatives of the following functions. (It is not necessary to simplify).

(a) $y = \tan^{-1}(x^3 + 1)$

NPC

$\boxed{\frac{1}{(x^3+1)^2+1} 3x^2}$ ④

(b) $F(x) = \sin^{-1} \sqrt{x}$

$\boxed{\frac{1}{\sqrt{1-(\sqrt{x})^2}} \frac{1}{2\sqrt{x}}}$ ④

(c) $y = x^{\ln x} = e^{\ln x^{\ln x}} = e^{(\ln x)^2}$

$\frac{dy}{dx} = e^{(\ln x)^2} (2 \ln x) \frac{1}{x} = x^{\ln x} \frac{2 \ln x}{x}$

or

$\boxed{x^{\ln x} \frac{2\ln x}{x}}$ ④

(8) 7. Use a linear approximation to estimate $\sqrt{99.5}$

$f(x) \approx f(a) + f'(a)(x-a)$, for $x$ near $a$

$f(x) = \sqrt{x}$ ② $a = 100$ ②, $f(100) = 10$, $f'(x) = \frac{1}{2\sqrt{x}}$, $f'(100) = \frac{1}{20}$

$\sqrt{x} \approx 10 + \frac{1}{20}(x - 100)$ for $x$ near $100$.

$\sqrt{99.5} \approx 10 + \frac{1}{20}(-0.5) = 10 - \frac{0.25}{10} = 10 - 0.025$

$= 9.975$ ④

$\boxed{9.975}$ 8

(6) 8. If a ball is thrown vertically upward with a velocity of 80 ft/sec, then its height after $t$ seconds is $s = 80t - 16t^2$. Find the acceleration of the ball when it reaches its maximum height.

$v = 80 - 32t$

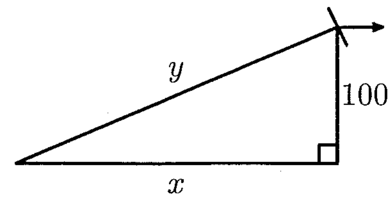
$a = -32$

NPC

$\boxed{-32 \text{ ft/sec}^2}$ 6

(14) 9. A kite 100 ft above the ground is being blown away from a person lying on the ground and holding its string. The kite moves parallel to the ground at a constant height and in a fixed direction, at the rate of 10 ft/sec. At what rate must the string be let out when the length of the string that is already let out is 200 ft?

$\frac{dx}{dt} = 10$ ft/sec (2)




Find $\frac{dy}{dt}$ when $y=200$ ft

$y^2 = x^2 + 100^2$ (4)

$2y\frac{dy}{dt} = 2x\frac{dx}{dt}$ (4)

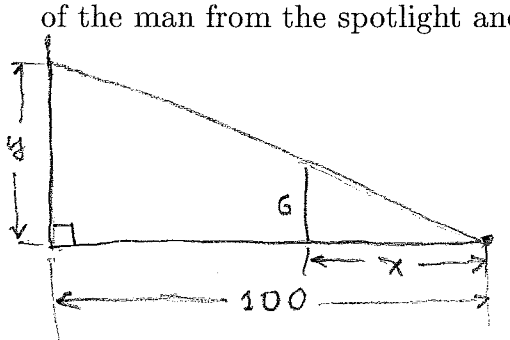
$\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$

When $y=200$: $x = \sqrt{200^2-100^2} = \sqrt{40000-10000} = \sqrt{30000} = 100\sqrt{3}$

$\frac{dy}{dt} = \frac{100\sqrt{3}}{200}\cdot 10 = 5\sqrt{3}$ ft/sec (4)

$\boxed{5\sqrt{3}\text{ ft/sec}}$ 14

(14) 10. A spotlight on the ground shines on a wall 100 ft away. A man 6 ft tall starts at the spotlight and walks directly towards ~~to~~ the wall at 5 ft/sec. How fast is the length of his shadow on the wall decreasing when he is 50 ft from the wall? (Let $x$ be the distance of the man from the spotlight and let $y$ be the length of his shadow on the wall).




$\frac{dx}{dt} = 5$ ft/sec (2)

Find $\frac{dy}{dt}$ when $x=50$ft

$\frac{y}{100} = \frac{6}{x}$ (4)

$y = \frac{600}{x}$

$\frac{dy}{dt} = -\frac{600}{x^2}\frac{dx}{dt}$ (4)

When $x=50$: $\frac{dy}{dt} = -\frac{600}{2500}\cdot 5 = -\frac{6}{5}$ ft/sec (4)

The length of his shadow is decreasing at the rate of $\boxed{\frac{6}{5}\text{ ft/sec}}$ 14