NAME GRADING KEY

10-digit PUID ______

RECITATION INSTRUCTOR ______

RECITATION TIME ______

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Page 2	/40
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Page 4	/24
TOTAL	/100

DIRECTIONS

1. Write your name, 10-digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit. Please write neatly. Remember, if we cannot read your work and follow it logically, you may receive no credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes, calculators, or any electronic devices may be used on this exam.

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(10) 1. Find the absolute maximum and absolute minimum values of the function $f(x) = (x^2 - 3)e^{-x}$ on the interval $[0, 4]$. ③

$$f'(x) = (x^2-3)e^{-x}(-1) + 2x e^{-x} = e^{-x}(-x^2+3+2x)$$

③

$$f'(x) = 0 \; ; \; -e^{-x}(x^2-2x-3) = 0 \longrightarrow (x-3)(x+1) = 0 \longrightarrow x = 3, -1$$

$x = -1$ not in $(0,4)$

$f(0) = -3$

$f(3) = 6e^{-3} = \frac{6}{e^3}$

$f(4) = 13e^{-4} = \frac{13}{e^4}$

$$\frac{13}{e^4} \overset{?}{<} \frac{6}{e^3} \rightarrow \frac{13}{e} \overset{?}{<} 6 \rightarrow \frac{13}{6} \overset{?}{<} e \text{ yes!}$$

②

abs. max. $\boxed{f(3) = \frac{6}{e^3}}$

abs. min. $\boxed{f(0) = -3}$

②

$\boxed{10}$

(10) 2. Find all numbers c that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^3 - 6x$ on the interval $[-2, 2]$.

$$f'(c) = \frac{f(2) - f(-2)}{2 - (-2)} \text{ ③ for some } c \in (-2, 2)$$

$$f'(c) = \frac{(8-12) - (-8+12)}{4} = \frac{-4-4}{4} = -2$$

$f'(x) = 3x^2 - 6$

$$3c^2 - 6 = -2 \longrightarrow c^2 = \frac{4}{3} \longrightarrow c = \pm\frac{2}{\sqrt{3}}$$

③

or

② ②

$\boxed{\frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}}}$

$\boxed{10}$

Name: ______________________________

(30) 3. Find each of the following as a real number, $+\infty$, $-\infty$, or write DNE (does not exist).

(a) $\lim_{x\to 0} \frac{x+\sin x}{x+\cos x}. = \frac{0}{1} = 0$

6 pts each
NPC

$\boxed{0}$ $\boxed{6}$

(b) $\lim_{x\to(\frac{\pi}{2})^+} \frac{\cos x}{1-\sin x} \overset{L'H}{=} \lim_{x\to(\frac{\pi}{2})^+} \frac{-\sin x}{-\cos x} = \lim_{x\to(\frac{\pi}{2})^+} \tan x = -\infty$

$\frac{0}{0}$

$\boxed{-\infty}$ $\boxed{6}$

(c) $\lim_{x\to 0} \frac{1-\cos x}{x^2} \overset{L'H}{=} \lim_{x\to 0} \frac{\sin x}{2x} = \frac{1}{2} \lim_{x\to 0} \frac{\sin x}{x} = \frac{1}{2}\cdot 1 = \frac{1}{2}$

$\frac{0}{0}$

$\boxed{\frac{1}{2}}$ $\boxed{6}$

(d) $\lim_{x\to\infty} \frac{x}{(\ln x)^2} \overset{L'H}{=} \lim_{x\to\infty} \frac{1}{2(\ln x)\frac{1}{x}} = \lim_{x\to\infty} \frac{x}{2\ln x} =$

$\frac{\infty}{\infty}$ $\frac{\infty}{\infty}$

$\overset{L'H}{=} \lim_{x\to\infty} \frac{1}{2\cdot\frac{1}{x}} = \lim_{x\to\infty} \frac{x}{2} = \infty$

$\boxed{\infty}$ $\boxed{6}$

(e) $\lim_{x\to 0^+} (1+2x)^{\frac{3}{x}} = \lim_{x\to 0^+} e^{\frac{3}{x}\ln(1+2x)} = e^{\lim_{x\to 0^+} \frac{3}{x}\ln(1+2x)} = e^6$

$\lim_{x\to 0^+} \frac{3\ln(1+2x)}{x} \overset{L'H}{=} \lim_{x\to 0^+} \frac{3\frac{1}{1+2x}\cdot 2}{1} = 6$

$\frac{0}{0}$

$\boxed{e^6}$ $\boxed{6}$

(10) 4. If 1200cm^2 of material is available to make a box with square base and an open top, find the largest possible volume of the box. (Let x be the length of an edge of the base and y be the height).

$x^2 + 4xy = 1200$ ③ $\longrightarrow y = \frac{1200-x^2}{4x}$

$V = x^2 y$

$V = x^2 \frac{1200-x^2}{4x} \longrightarrow V = 300x - \frac{x^3}{4}$ ③, $0 < x < 10\sqrt{12}$

$\frac{dV}{dx} = 300 - \frac{3}{4}x^2$

$\frac{dV}{dx} = 0 \rightarrow x^2 = 400 \rightarrow x = 20$ ②

$\frac{dV}{dx}$: 0 + + + + + + $\overset{0}{20}$ − − − $10\sqrt{12}$; 20 max

$\max V = 300\cdot 20 - \frac{400\cdot 20}{4} = 6000 - 2000 = 4000$

② $\boxed{4000\text{ cm}^3}$ $\boxed{10}$

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(16) 5. Let $f(x) = \dfrac{2-x^2}{(x+1)^2}$. Give all the requested information and sketch the graph of the function on the axes below. Give both coordinates of the intercepts, local extrema and points of inflection, and give an equation for each asymptote. Write NONE where appropriate.

Domain: all $x \neq -1$

x-int.: $y=0 \rightarrow 2-x^2=0 \rightarrow x = \pm\sqrt{2}$

y-int.: $x=0 \rightarrow y=2$

Symmetry — none

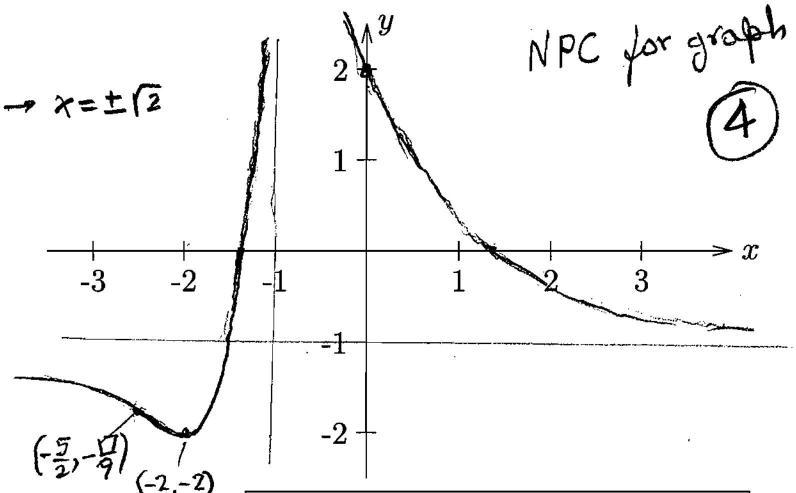
V.A.: $\lim\limits_{x\to(-1)^+} \dfrac{2-x^2}{(x+1)^2} = \infty$

$\lim\limits_{x\to(-1)^-} \dfrac{2-x^2}{(x+1)^2} = \infty$

Vert. As. $x=-1$

H.A: $\lim\limits_{x\to\pm\infty} \dfrac{2-x^2}{(x+1)^2} = -1$

Hor. As. $y=-1$



NPC for graph ④

$$f'(x) = \frac{(x+1)^2(-2x)-(2-x^2)2(x+1)}{(x+1)^4}$$

$$= \frac{-2x^2-2x-4+2x^2}{(x+1)^3}$$

$$= -\frac{2(x+2)}{(x+1)^3}$$

$f'(x)$ - - - 0 + + + | - - - - - x

-2 -1

↑ loc min

$f(-2) = -2$

$$f''(x) = -\frac{(x+1)^3 2 - 2(x+2)3(x+1)^2}{(x+1)^6}$$

$$= -\frac{2x+2-6x-12}{(x+1)^4}$$

$$= -\frac{-4x-10}{(x+1)^4}$$

$$= \frac{2(2x+5)}{(x+1)^4}$$

$f''(x)$ - - - 0 + + + | + + + + x

$-\frac{5}{2}$ -1

domain	$(-\infty,-1) \cup (-1,\infty)$	①
intercepts	$(-\sqrt{2},0)$ $(\sqrt{2},0)$ $(0,2)$	①
symmetry	NONE	①
horizontal asymptotes	$y=-1$	①
vertical asymptotes	$x=-1$	①
intervals of increase	$(-2,-1)$	①
intervals of decrease	$(-\infty,-2)$ $(-1,\infty)$	①
local maxima	NONE	①
local minima	$(-2,-2)$	①
intervals of concave down	$(-\infty,-\frac{5}{2})$	①
intervals of concave up	$(-\frac{5}{2},-1)$ $(-1,\infty)$	①
points of inflection	$(-\frac{5}{2},-\frac{17}{9})$	①

$$f(-\tfrac{5}{2}) = \frac{2-(-\frac{5}{2})^2}{(-\frac{3}{2})^2} = \frac{2-\frac{25}{4}}{\frac{9}{4}} = \frac{8-25}{9} = -\frac{17}{9}$$

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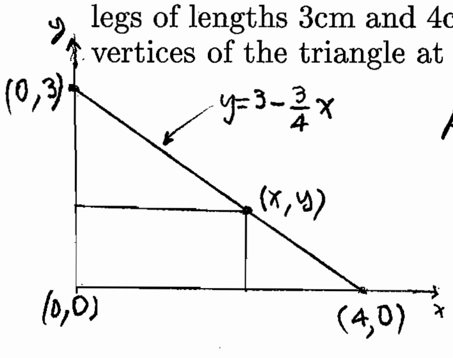
(10) 6. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3cm and 4cm if two sides of the rectangle lie along the legs. (Place the vertices of the triangle at the points $(0,0)$, $(4,0)$ and $(0,3)$ of the (x,y)–plane).



$A = xy$ $\quad y = 3-\frac{3}{4}x$ ③

$A = x\left(3-\frac{3}{4}x\right)$

$A = 3x - \frac{3}{4}x^2$ ③ $\quad 0 \le x \le 4$

$\frac{dA}{dx} = 3-\frac{3}{2}x$

$\frac{dA}{dx}=0 \rightarrow x=2$ ②

$A(2) = 2\left(3-\frac{3}{4}\cdot 2\right) = 3$ ②

$\frac{dA}{dx}$ $\underset{2}{+++\,0\,---}$ ↖ max

$\boxed{3\text{ cm}^2}$ $\quad \boxed{10}$

(5) 7. A particle moves in a straight line and has acceleration given by $a(t) = \sin t$. Its initial velocity is $v(0) = 0$ and its initial position is $s(0) = 0$. Find its position function $s(t)$.

$v'(t) = a(t) = \sin t$

$v(t) = -\cos t + C$

$t=0:\; 0 = -1 + C \longrightarrow C = 1$

$v(t) = -\cos t + 1$

$s'(t) = v(t) = -\cos t + 1$

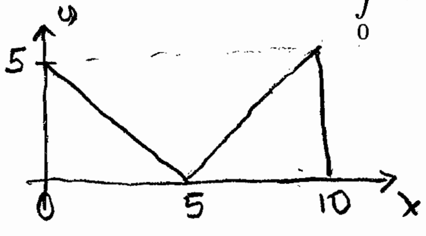
$s(t) = -\sin t + t + D$

$t=0:\; 0 = 0 \quad 0 + D \rightarrow D=0$

NPC

$\boxed{s(t) = -\sin t + t}$ $\quad \boxed{5}$

(5) 8. Evaluate the integral $\int_0^{10} |x-5|dx$ by interpreting it in terms of areas.



$\int_0^{10} |x-5|dx = \frac{1}{2}\cdot 5\cdot 5 + \frac{1}{2}\cdot 5\cdot 5 = 25$

NPC

$\boxed{25}$ $\quad \boxed{5}$

(4) 9. Find the most general antiderivative of $f(x) = \sqrt[4]{x^3} + \sqrt[3]{x^4}$.

$f(x) = x^{\frac{3}{4}} + x^{\frac{4}{3}}$

$F(x) = \frac{x^{7/4}}{\frac{7}{4}} + \frac{x^{7/3}}{\frac{7}{3}} + C$ ↖ or →

-1pt for missing +C

$\boxed{\frac{4}{7}x^{\frac{7}{4}} + \frac{3}{7}x^{\frac{7}{3}} + C}$ $\quad \boxed{4}$