NAME GRADING KEY

STUDENT ID ______

RECITATION INSTRUCTOR ______

RECITATION TIME ______

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DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, and 4
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this test.

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(10) 1. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three-dimensional vectors. For each statement below, circle T if the statement is always true, or F if it is not always true.

2 pts each

(i) $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$ T **F**

(ii) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ **T** F

(iii) $(\vec{a} \times \vec{b}) \times \vec{c}$ is a real number T **F**

(iv) $(\vec{a} \times \vec{b}) \cdot \vec{c}$ is a vector T **F**

(v) If $\vec{a} \cdot \vec{b} = \vec{c} \cdot \vec{b}$, then $\vec{a} = \vec{c}$ T **F**

In problems 2-5, if arrows indicating vectors are missing, put them in. If more than 2 arrows are missing in 2-5, take a point off.

(7) 2. Find a unit vector that is perpendicular to both $\vec{i} + \vec{j}$ and $\vec{j} - \vec{k}$.

$$(\vec{i}+\vec{j}) \times (\vec{j}-\vec{k}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = \underbrace{-\vec{i}+\vec{j}+\vec{k}}_{-2\text{ pts for 1 wrong coeff.}} \quad (4)$$

$$|-\vec{i}+\vec{j}+\vec{k}| = \sqrt{3}$$

$$\vec{u} = \frac{1}{\sqrt{3}}(-\vec{i}+\vec{j}+\vec{k}) \quad (3)$$

$\boxed{\frac{1}{\sqrt{3}}(-\vec{i}+\vec{j}+\vec{k})}$ $\boxed{7}$

(10) 3. Find the area of the triangle PQR with vertices at $P(1,0,-1)$, $Q(1,2,1)$, and $R(0,1,1)$.

$\vec{PQ} = 2\vec{j} + 2\vec{k} \qquad \vec{PR} = -\vec{i} + \vec{j} + 2\vec{k}$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 2 \\ -1 & 1 & 2 \end{vmatrix} = (4-2)\vec{i} - [0\cdot 2 - 2(-1)]\vec{j} + [0\cdot 1 - 2(-1)]\vec{k}$$

$$= \underbrace{2\vec{i} - 2\vec{j} + 2\vec{k}}_{-2\text{ pts for 1 wrong coeff.}} \quad (4)$$

Area of $PQR = \frac{1}{2}|\vec{PQ} \times \vec{PR}|$ (4)

$= \frac{1}{2}\sqrt{12} = \sqrt{3}$ or (2)

$\boxed{\sqrt{3}}$ 10

(8) 4. If $\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k}$ and $\vec{b} = \vec{i} - \vec{j}$, find the vector projection of $\vec{b}$ onto $\vec{a}$, $\text{proj}_{\vec{a}}\vec{b}$.

$\text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a}$ (4)

$= \frac{2+3}{4+9+1}(2\vec{i} - 3\vec{j} + \vec{k})$

$= \frac{5}{14}(2\vec{i} - 3\vec{j} + \vec{k})$ (4)

$\boxed{\frac{5}{14}(2\vec{i} - 3\vec{j} + \vec{k})}$ 8

(5) 5. Find the value of $x \neq 0$ such that the vectors $\langle -3x, 2x\rangle$ and $\langle 4, x\rangle$ are orthogonal.

$\langle -3x, 2x\rangle \cdot \langle 4, x\rangle = 0$ (3)

$-12x + 2x^2 = 0$

$x(x-6) = 0$

$x = 6$ (2)

$\boxed{6}$ 5

(8) 6. Find an equation of the sphere that passes through the origin and whose center is $(1,2,3)$

$(x-1)^2 + (y-2)^2 + (z-3)^2 = a^2$ (4)

$(x,y,z) = (0,0,0)$ lies on the sphere:

$(-1)^2 + (-2)^2 + (-3)^2 = a^2$

$a^2 = 14$ or (4)

or $a = \sqrt{(1-0)^2 + (2-0)^2 + (3-0)^2} = \sqrt{14}$

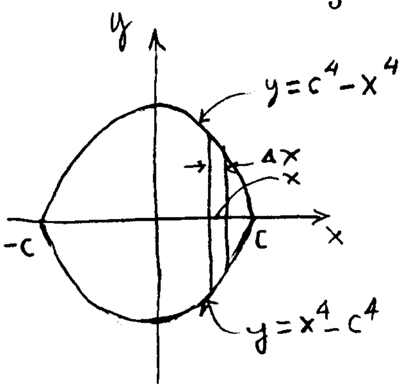
-2 pts if $\sqrt{14}$

(10) 7. Find the value of the positive number c such that the area of the region enclosed by the curves

$$y = x^4 - c^4 \text{ and } y = c^4 - x^4$$

is equal to $\frac{16}{5}$.

* 0 credit for problem if more than 1 item is wrong (limits count as 1 item in this rule)



Area of typical approximating rectangle

$$\Delta A = \left[(c^4-x^4)-(x^4-c^4)\right]\Delta x \quad ①$$

$$A = \int_{-c}^{c} \underbrace{(2c^4-2x^4)}_{⑤}\, \underbrace{dx}_{①} * \qquad ① \qquad \left(\text{or } A = 4\int_0^c (c^4-x^4)\,dx \text{ from symmetry}\right)$$

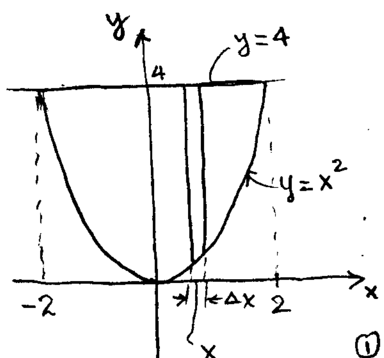
$$A = \left(2c^4x - 2\frac{x^5}{5}\right)\Big|_{-c}^{c} = \left(2c^5 - 2\frac{c^5}{5}\right) - \left(-2c^5 + 2\frac{c^5}{5}\right)$$

$$= 4c^5 - \frac{4}{5}c^5 = \frac{16}{5}c^5$$

$$\frac{16}{5}c^5 = \frac{16}{5} \quad \therefore c = 1 \quad ②$$

$\boxed{1}$ $\boxed{10}$

(8) 8. Set up an integral for the volume of the solid obtained by rotating the region bounded by the curves $y = x^2$ and $y = 4$ about the line $y = 4$. Do not evaluate the integral.



Volume of approximating disk:

$$\Delta V = \pi(4-x^2)^2\Delta x$$

$$V = \int_{-2}^{2} \pi(4-x^2)^2\,dx$$

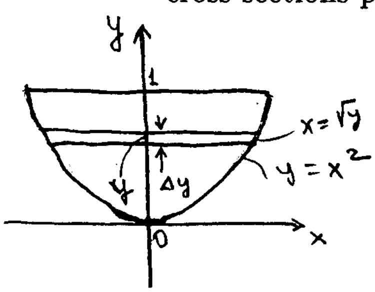
↑ *

Or by shells: $V = \int_{0\,①}^{4\,①} \underbrace{2\pi(4-y)\,2\sqrt{y}}_{⑤}\, dy \; ①$

$$\boxed{V = \int_{-2}^{2} \overbrace{\pi(4-x^2)^2}^{5}\,dx}$$

① ① ① $\boxed{8}$

(8) 9. The base of the solid S is the region bounded by the curves $y = x^2$ and $y = 1$, and cross-sections perpendicular to the y-axis are squares. Find the volume of S.



Volume of typical approximating slice:

$$\Delta V = (2\sqrt{y})^2\Delta y \quad ①$$

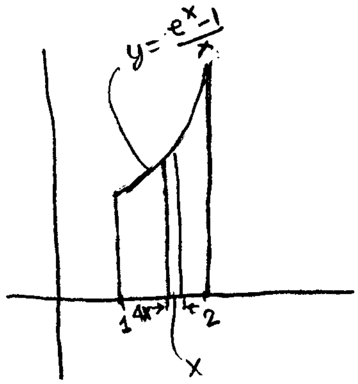
↑

$$V = \int_0^1 \underbrace{(2\sqrt{y})^2}_{④}\, dy \; ① \qquad ① \qquad *$$

$$= 2y^2\Big|_0^1$$

$$= 2 \quad ①$$

(10) 10. The region bounded by the curves $y = \dfrac{e^x - 1}{x}$, $x = 1$, $x = 2$, and $y = 0$ is rotated about the y-axis. Find the volume of the solid thus obtained.



Volume of typical approx. shell:

$$\Delta V = 2\pi x \, \frac{e^x - 1}{x} \, \Delta x$$

$$V = \int_1^2 2\pi (e^x - 1)\, dx$$

① ① ⑤ ①

* see p 3

$$= 2\pi \, (e^x - x)\Big|_1^2$$

$$= 2\pi \, (e^2 - 2 - e + 1)$$

$$= 2\pi \, (e^2 - e - 1) \quad ②$$

$$\boxed{2\pi(e^2 - e - 1)}$$

(8) 11. If the work required to stretch a spring 1 ft beyond its natural length is 12 ft-lb, how much work is needed to stretch it 9 in beyond its natural length?

$$\int_0^1 kx\, dx = 12$$

$$\frac{kx^2}{2}\Big|_0^1 = 12$$

$$\frac{k}{2} = 12 \rightarrow k = 24 \quad ④$$

$$W = \int_0^{\frac{9}{12}} 24x\, dx =$$

-2 pts if this is 9 and answer is $12 \cdot 81 = 972$

$$= 12x^2\Big|_0^{3/4} = 12 \cdot \frac{9}{16} = \frac{27}{4} \quad ④$$

$$\boxed{\frac{27}{4} \text{ ft-lbs}}$$

$\boxed{8}$

(8) 12. Evaluate the integral $\int x^2 \ln x\, dx$.

$$\int x^2 \ln x\, dx = \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\frac{1}{x}\, dx = \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2\, dx$$

$u = \ln x \qquad dv = x^2\, dx$

$du = \frac{1}{x}\, dx \qquad v = \frac{x^3}{3}$

$$= \frac{x^3}{3}\ln x - \frac{x^3}{9} + C$$

④ ④