NAME GRADING KEY

10–DIGIT PUID ____________

RECITATION INSTRUCTOR ____________

RECITATION TIME ____________

Page 1	/20
Page 2	/23
Page 3	/26
Page 4	/31
TOTAL	/100

DIRECTIONS

1. Write your name, 10–digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this test.

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(10) 1. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three–dimensional vectors. For each statement below, circle T if the statement is always true, or F if it is not always true. 2pts each

(i) $\vec{a}\cdot\vec{a} = |\vec{a}|^2$ T F

(ii) $\vec{a}\cdot(\vec{b}\cdot\vec{c}) = (\vec{a}\cdot\vec{b})\cdot\vec{c}$ Since $\vec{b}\cdot\vec{c}$ is a scalar, $\vec{a}\cdot(\vec{b}\cdot\vec{c})$ has no meaning T F

(iii) $\vec{a}\cdot(\vec{b}+\vec{c}) = (\vec{a}+\vec{b})\cdot\vec{c}$ T F

(iv) $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$ T F

(v) If $\vec{a}\times\vec{c} = \vec{b}\times\vec{c}$, then $\vec{a}=\vec{b}$ T F $\boxed{10}$

(6) 2. For what values of b are the vectors $< 2,-1,b >$ and $< b^2,3,b >$ orthogonal?

$\langle 2,-1,b\rangle\cdot\langle b^2,3,b\rangle = 0$ 3

$2b^2-3+b^2=0$

$b^2=1 \Rightarrow b=\pm 1$ 3

$\boxed{b = \pm 1}$ $\boxed{6}$

(4) 3. Find $\vec{a}\cdot\vec{b}$ if $|\vec{a}|=3$, $|\vec{b}|=6$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$ radians.

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta = 3\cdot 6\cdot\frac{1}{2} = 9$ NPC

$\boxed{\vec{a}\cdot\vec{b} = 9}$ $\boxed{4}$

0 pts for $18\cos\frac{\pi}{3}$

(6) 4. Find a vector that has the same direction as $< -2, 4, 2 >$ but has length 6.

$$|<-2,4,2>| = \sqrt{4+16+4} = \sqrt{24}$$

NPC

$\frac{1}{\sqrt{24}} <-2,4,2>$ is a unit vector in the direction of $<-2,4,2>$

$\frac{6}{\sqrt{24}} <-2,4,2>$ is the desired vector

$$\boxed{\frac{6}{\sqrt{24}} <-2,4,2>}$$

$\boxed{6}$

(4) 5. Are the vectors $< 1, -2, 3 >$ and $< 3, -6, 9 >$ orthogonal, parallel or neither?

$<3,-6,9> = 3<1,-2,3>$

$\therefore$ the vectors are parallel

NPC

$\boxed{\text{parallel}}$

$\boxed{4}$

(13) 6. Consider the three points $A(1,1,1), B(2,0,2), C(1,1,2)$.

(a) Find $\vec{AB} \times \vec{AC}$

$$\vec{AB} = \vec{i} - \vec{j} + \vec{k}$$

$$\vec{AC} = \vec{k}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = -\vec{i} - \vec{j}$$

5

$\vec{AB} \times \vec{AC} = -\vec{i} - \vec{j}$

Grade (b) and (c) consistently with answer in (a).

(b) Find the area of the triangle with vertices A, B, C.

$$\frac{1}{2} |\vec{AB} \times \vec{AC}| = \frac{1}{2}\sqrt{2}$$

4

$$\boxed{\frac{\sqrt{2}}{2}}$$

(c) Find a unit vector orthogonal to the plane that passes through the points A, B, C.

$$\frac{1}{\sqrt{2}}(-\vec{i} - \vec{j})$$

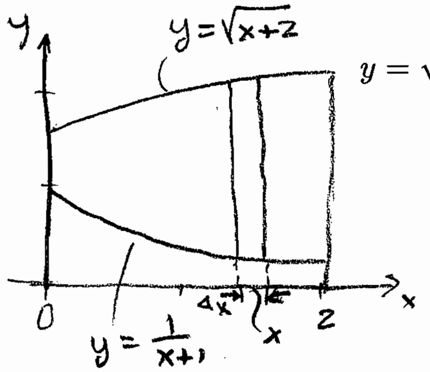
4

$$\boxed{\frac{1}{\sqrt{2}}(-\vec{i} - \vec{j})}$$

$\boxed{13}$

(10) 7. Find the area of the region bounded by the curves

$$y = \sqrt{x+2}, \quad y = \frac{1}{x+1}, \quad x = 0, \quad x = 2.$$



Area of typical approximating rectangle:

$$\Delta A = \left(\sqrt{x+2} - \frac{1}{x+1}\right)\Delta x$$

$$A = \int_0^2 \underbrace{\left(\sqrt{x+2} - \frac{1}{x+1}\right)}_{5} dx$$

1 1 1

Rule*: $\boxed{\text{0 points for problem if more than 1 item is wrong. (limits count as 1 item in this rule)}}$

$$A = \left[\frac{2}{3}(x+2)^{3/2} - \ln(x+1)\right]_0^2$$

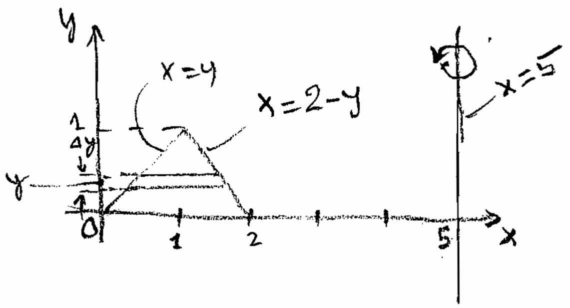
$$= \left(\frac{2}{3}\cdot 4^{3/2} - \ln 3\right) - \left(\frac{2}{3}\cdot 2^{3/2} - \ln 1\right) = \boxed{\frac{16}{3} - \ln 3 - \frac{4}{3}\sqrt{2}}$$

2

$\boxed{10}$

(16) 8. Let R be the region bounded by $y = x$, $y = 2 - x$, and $y = 0$.

(a) Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating R about the line $x = 5$, using the method of disks/washers.



Volume of typical washer

$$\left[\pi(5-y)^2 - \pi\left(5-(2-y)\right)^2\right]\Delta y$$

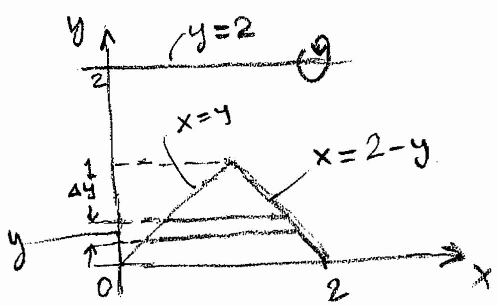
Rule *

$$\boxed{V = \int_0^1 \overbrace{\left[\pi(5-y)^2 - \pi\left(5-(2-y)\right)^2\right]}^{5} dy}$$

1 1 1

$\boxed{8}$

(b) Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating R about the line $y = 2$, using the method of cylindrical shells.



Volume of typical shell

$$\Delta V = 2\pi(2-y)\left[(2-y) - y\right]\Delta y$$

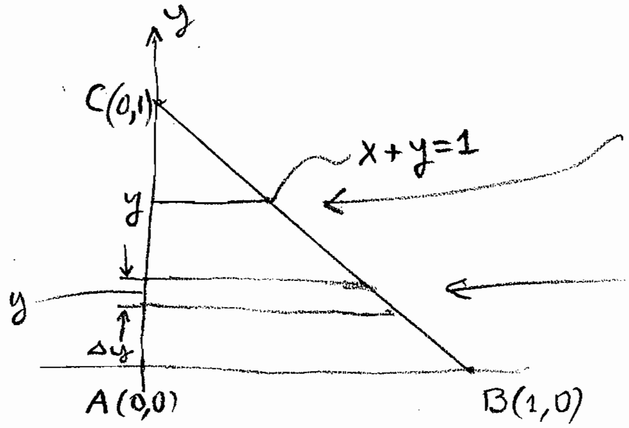
Rule *

$$\boxed{V = \int_0^1 \overbrace{2\pi(2-y)\left[(2-y) - y\right]}^{5} dy}$$

1 1 1

$\boxed{8}$

(10) 9. The base of a solid is a triangular region with vertices $A(0,0)$, $B(1,0)$, and $C(0,1)$. Cross–sections perpendicular to the y–axis are semicircles. Find the volume of the solid.



Area of typical cross section:

$$A(y) = \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 = \frac{1}{2}\pi\left(\frac{1-y}{2}\right)^2 = \frac{\pi}{8}(1-y)^2$$

Volume of typical slice:

$$\Delta V = A(y)\Delta y = \frac{\pi}{8}(1-y)^2\Delta y$$

$$V = \int_0^1 \underbrace{\frac{\pi}{8}(1-y)^2}_{5} dy \qquad \text{Rule } *$$

$$V = \int_0^1 \frac{\pi}{8}(1-2y+y^2)\,dy =$$

$$= \frac{\pi}{8}\left(y - y^2 + \frac{y^3}{3}\right)\Big|_0^1 = \frac{\pi}{8}\left(1-1+\frac{1}{3}\right) = \frac{\pi}{24}$$

$\boxed{V = \frac{\pi}{24}}$ $\boxed{10}$

(6) 10. Find the average value of $f(x) = \sqrt{x}$ on the interval $[0,4]$.

$$f_{ave} = \frac{1}{4}\int_0^4 \sqrt{x}\,dx = \frac{1}{4}\,\frac{x^{3/2}}{\frac{3}{2}}\Big|_0^4$$

$$= \frac{1}{6}\,4^{3/2} = \frac{8}{6} = \frac{4}{3} \quad 3$$

$\boxed{\frac{4}{3}}$ $\boxed{6}$

(15) 11. Evaluate the integrals

(a) $\int x^3 \ln x dx$ $\underset{\substack{u=\ln x \quad dv=x^3dx \\ du=\frac{1}{x}dx \quad v=\frac{x^4}{4}}}{=\!=\!=}$ $\overbrace{\frac{x^4}{4}\ln x}^{2} - \overbrace{\int \frac{x^4}{4}\frac{1}{x}dx}^{2} =$

$$= \frac{x^4}{4}\ln x - \int \frac{x^3}{4}dx = \frac{x^4}{4}\ln x - \frac{x^4}{16} + C$$

$\boxed{\frac{x^4}{4}\ln x - \frac{x^4}{16} + C}$ $\boxed{9}$

(b) $\int_0^\pi t\sin 3tdt$ $\underset{\substack{u=t,\ dv=\sin 3t dt \\ du=dt,\ v=-\frac{1}{3}\cos 3t}}{=\!=\!=}$ $\overbrace{-\frac{1}{3}t\cos 3t\Big|_0^\pi}^{2} + \overbrace{\int_0^\pi \frac{1}{3}\cos 3t\,dt}^{2} =$

$$= -\frac{1}{3}\pi\cos 3\pi + \left[\frac{1}{9}\sin 3t\right]_0^\pi = \frac{\pi}{3} \quad 2$$

$\boxed{\frac{\pi}{3}}$ $\boxed{6}$