1. Find the center and radius of the sphere with equation:

\[ 2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1 \]

\[ 2\left(x^2 - 4x\right) + 2y^2 + 2\left(z^2 + 12z\right) = 1 \]

\[ 2\left(x^2 - 4x + 4\right) + 2y^2 + 2\left(z^2 + 12z + 36\right) = 1 + 8 + 72 \]

\[ 2\left(x - 2\right)^2 + 2y^2 + 2\left(z + 6\right)^2 = \frac{81}{2} \]

\[ (x-2)^2 + y^2 + (z+6)^2 = \frac{81}{2} < \text{center (2, 0, -6)} \]

\[ \text{radius } \frac{9 \sqrt{2}}{2} \]

2. If \( \vec{a} = \langle 5, -12 \rangle \) and \( \vec{b} = \langle -3, -6 \rangle \), find the following:

\[ \vec{a} + \vec{b} = \langle 5 - 3, -12 - 6 \rangle = \langle 2, -18 \rangle \]

\[ 2\vec{a} + 3\vec{b} = \langle 10, -24 \rangle + \langle -9, -18 \rangle = \langle 1, -42 \rangle \]

\[ |\vec{a}| = \sqrt{(5)^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13 \]

\[ |\vec{a} - \vec{b}| = |\langle 8, -6 \rangle| = \sqrt{64 + 36} = \sqrt{100} = 10 \]

\[ \vec{a} + \vec{b} = \langle 2, -18 \rangle \]

\[ 2\vec{a} + 3\vec{b} = \langle 1, -42 \rangle \]

\[ |\vec{a}| = 13 \]

\[ |\vec{a} - \vec{b}| = 10 \]
(8) 3. Find all values of \( t \) for which the angle between the vectors \( \vec{a} = t \vec{i} + \vec{j} + \vec{k} \) and \( \vec{b} = \vec{i} + t \vec{j} + \vec{k} \) is \( \frac{\pi}{4} \).
\[
\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \frac{\pi}{4} \quad \Rightarrow \quad \frac{t}{\sqrt{t^2+2}} \sqrt{t^2+2} \cos \frac{\pi}{4} = 2 \frac{t+1}{2} - 4t=0 \rightarrow \quad t = 0, 4
\]

(8) 4. Let \( \vec{a} = (1, 2, 1) \) and \( \vec{b} = (3, -4, 2) \).

(i) Find all unit vectors \( \vec{u} \) that are perpendicular to both \( \vec{a} \) and \( \vec{b} \).
\[
\vec{a} \times \vec{b} = \left| \begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 3 & -4 & 2 \end{array} \right| = 8 \vec{i} + 10 \vec{j} + 10 \vec{k}
\]

\[
|\vec{a} \times \vec{b}| = \sqrt{64 + 100} = \sqrt{165}
\]

If \( \vec{a} \times \vec{b} \) is wrong, grade the rest of the problem using consistency with student's answer.

(ii) The area \( A \) of the parallelogram determined by \( \vec{a} \) and \( \vec{b} \) is
\[
A = \sqrt{165}
\]

(8) 5. Let \( \vec{a} = (6, 3, 1) \), \( \vec{b} = (0, 1, 2) \), \( \vec{c} = (4, -2, 5) \).

(i) The scalar triple product \( \vec{a} \cdot (\vec{b} \times \vec{c}) \) is
\[
\vec{b} \times \vec{c} = \left| \begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 2 \\ 4 & -2 & 5 \end{array} \right| = -9 \vec{i} + 8 \vec{j} - 4 \vec{k}
\]
\[
\vec{a} \cdot (\vec{b} \times \vec{c}) = 54 + 24 - 4 = 74
\]

(ii) True or False:

The vectors \( \vec{a}, \vec{b}, \vec{c} \) are coplanar. (circle one)

(8) 6. Set up, but do not evaluate, an integral for the area \( A \) of the region bounded by the curves \( y = \ln x \), \( x = y^2 - 2 \), \( y = -1 \), \( y = 1 \).

\[x = y^2 - 2\]
\[x = e^y\]

\[\text{Area of typical approximating rectangle:}\]
\[
\Delta A = \left[ e^y - (y^2 - 2) \right] dy
\]

\[\text{Rule X:}\]
\[0 \text{ pts for problem if more than 1 item is wrong. Limit count as 1 item in this rule.}\]
\[A = \int_{1}^{4} \left[ e^y - (y^2 - 2) \right] dy\]
7. Let $R$ be the region bounded by the curves $y = x^2$ and $y = -2x$. Use the method of washers to set up an integral for the volume $V$ of the solid obtained by rotating $R$ about the $x$-axis. Do not evaluate the integral.

**Points of intersection:**
$$x^2 = -2x$$
$$x(x+2) = 0$$
$$x = -2, 0$$

**Volume of typical approximating washer:**
$$\Delta V = \left[ \pi (-2x)^2 - \pi (x^2)^2 \right] \Delta x$$

**Rule:**
$$V = \int_{-2}^{0} (\pi 4x^2 - \pi x^4) \, dx$$

8. Let $R$ be the region bounded by the curves $y = x^4$ and $y = x$. Use the method of cylindrical shells to set up an integral for the volume $V$ of the solid obtained by rotating $R$ about the line $y = -4$. Do not evaluate the integral.

**Volume of typical approximating cylindrical shell:**
$$\Delta V = 2\pi \left[ y - (-4) \right] \left[ x^{4/3} - y \right] \Delta y$$

**Rule:**
$$V = \int_{0}^{1} 2\pi (y+4)(y^{4/3} - y) \, dy$$

9. A force of 10 lb is required to hold a spring stretched 4 in. beyond its natural length. How much work is done in stretching it from its natural length to 6 in. beyond its natural length?

$$F = kx$$
$$F = 10\text{lb}$$
$$x = \frac{1}{3} \text{ft}$$
$$10 = k \frac{1}{3} \rightarrow k = 30$$

**Work:**
$$W = \int_{0}^{\frac{1}{3}} 30x \, dx = 15x^2 \bigg|_{0}^{\frac{1}{3}} = 15 \cdot \frac{1}{4} = \frac{15}{4} \text{ ft-lb}$$

-3 pts if inches are not changed to feet,

and the answers are incorrect using consistency

$$W = \frac{15}{4} \text{ ft-lbs}$$
(11) 10. An aquarium 5 ft long, 4 ft wide and 2 ft deep is full of water. Find the work needed to pump half of the water out of the aquarium. (Use the fact that water weighs 62.5 lb/ft³).

\[
\text{Volume of typical layer } = 20 \Delta y \\
\text{Weight } = (62.5)20 \Delta y \\
\text{Work needed for lifting typical layer from } y \text{ to } 2: \Delta W = (2-y)(62.5)20 \Delta y
\]

\[
W = \int \left( (2-y)(62.5)20 \right) dy \quad \text{(Rule of Integration)} \\
= \frac{1}{2} (4-2)-(2-\frac{1}{2}) = \frac{1}{2}
\]

(11) 12. First make a substitution \( x = \tan \theta \) and then use integration by parts to evaluate the integral \( \int \theta^3 \cos(\theta^2)d\theta \). Your answer in the box must be in terms of \( \theta \).

\[
\int \theta^3 \cos(\theta^2)d\theta = \frac{1}{2} \int \theta \cos \theta d\theta = \frac{1}{2} \left[ \theta \sin \theta + \cos \theta \right] + C
\]

\[
\int \cos \theta d\theta = \sin \theta \\
u = \theta \\
dv = \cos \theta d\theta \\
\int \sin \theta d\theta = -\cos \theta \\
= \frac{1}{2} \left[ \theta^2 \sin \theta - \frac{1}{2} \cos \theta \right] + C
\]

-2pin if \( \frac{1}{2} \) is missing in one or both terms

\[
\frac{1}{2} \theta^2 \sin(\theta^2) + \frac{1}{2} \cos(\theta^2) + C
\]