NAME GRADING KEY

10–DIGIT PUID ____________

RECITATION INSTRUCTOR ____________

RECITATION TIME ____________

Page 1	/14
Page 2	/32
Page 3	/24
Page 4	/30
TOTAL	/100

DIRECTIONS

1. Write your name, 10–digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes, calculators, or any electronic devices may be used on this test.

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(10) 1. Find the center and radius of the sphere with equation:

$2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1$

$2(x^2 - 4x \quad) + 2y^2 + 2(z^2 + 12z \quad) = 1$

$2(x^2 - 4x + 4) + 2y^2 + 2(z^2 + 12z + 36) = 1 + 8 + 72$

$2(x-2)^2 + 2y^2 + 2(z+6)^2 = 81$

$(x-2)^2 + y^2 + (z+6)^2 = \frac{81}{2}$ ← For 6

center $(2, 0, -6)$ 2

radius $\frac{9}{\sqrt{2}}$ 2

10

(4) 2. If $\vec{a} = \langle 5, -12 \rangle$ and $\vec{b} = \langle -3, -6 \rangle$, find the following:

$\vec{a} + \vec{b} = \langle 5-3, -12-6 \rangle = \langle 2, -18 \rangle$

1 pt each

$2\vec{a} + 3\vec{b} = \langle 10, -24 \rangle + \langle -9, -18 \rangle = \langle 1, -42 \rangle$

$|\vec{a}| = \sqrt{(5)^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$

$|\vec{a} - \vec{b}| = |\langle 8, -6 \rangle| = \sqrt{64 + 36} = \sqrt{100} = 10$

$\vec{a} + \vec{b} =$ $\langle 2, -18 \rangle$

$2\vec{a} + 3\vec{b} =$ $\langle 1, -42 \rangle$

$|\vec{a}| =$ 13

$|\vec{a} - \vec{b}| =$ 10

4

(8) 3. Find all values of t for which the angle between the vectors $\vec{a} = t\vec{i} + \vec{j} + \vec{k}$ and $\vec{b} = \vec{i} + t\vec{j} + \vec{k}$ is $\frac{\pi}{3}$.

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta \rightarrow t + t + 1 = \sqrt{t^2+2}\sqrt{t^2+2}\cos\frac{\pi}{3}$ (4)

$2t + 1 = (t^2+2)\frac{1}{2}$

$4t + 2 = t^2 + 2 \rightarrow t^2 - 4t = 0 \rightarrow t = 0, 4$

(2) (2)

$\boxed{t = 0, 4}$

8

(8) 4. Let $\vec{a} = \langle 1, 2, 1\rangle$ and $\vec{b} = \langle 3, -4, 2\rangle$.

(i) Find all unit vectors $\vec{u}$ that are perpendicular to both $\vec{a}$ and $\vec{b}$.

$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 3 & -4 & 2 \end{vmatrix} = 8\vec{i} + \vec{j} - 10\vec{k}$ (4)

$|\vec{a} \times \vec{b}| = \sqrt{64 + 1 + 100} = \sqrt{165}$

[If $\vec{a} \times \vec{b}$ is wrong, grade the rest of the problem using consistency with student's answer

(1)

$\vec{u} = \boxed{\begin{array}{c} \frac{1}{\sqrt{165}}(8\vec{i} + \vec{j} - 10\vec{k}) \\ -\frac{1}{\sqrt{165}}(8\vec{i} + \vec{j} - 10\vec{k}) \end{array}}$

(1)

(ii) The area A of the parallelogram determined by $\vec{a}$ and $\vec{b}$ is

$\boxed{A = \sqrt{165}}$ (2)

8

(8) 5. Let $\vec{a} = \langle 6, 3, 1\rangle$, $\vec{b} = \langle 0, 1, 2\rangle$, $\vec{c} = \langle 4, -2, 5\rangle$.

(i) The scalar triple product $\vec{a} \cdot (b \times c)$ is

$\vec{b} \times \vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 2 \\ 4 & -2 & 5 \end{vmatrix} = 9\vec{i} + 8\vec{j} - 4\vec{k}$

$\vec{a} \cdot (\vec{b} \times \vec{c}) = 54 + 24 - 4 = 74$

Or $\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 6 & 3 & 1 \\ 0 & 1 & 2 \\ 4 & -2 & 5 \end{vmatrix}$

$= 6 \cdot 9 - 3(-8) + (-4) =$

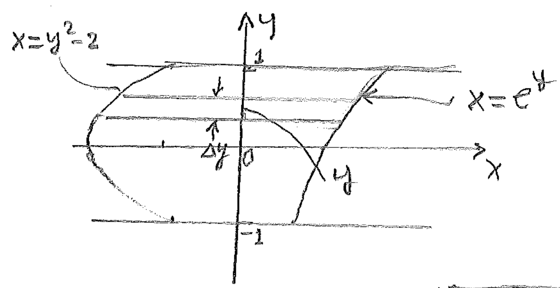
$\boxed{\vec{a} \cdot (\vec{b} \times \vec{c}) = 74}$ (5)

(ii) True or False:

The vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar. (circle one) T F (3)

8

(8) 6. Set up, but do not evaluate, an integral for the area A of the region bounded by the curves $y = \ln x$, $x = y^2 - 2$, $y = -1$, $y = 1$.



Area of typical approximating rectangle:

$\Delta A = [e^y - (y^2 - 2)]\Delta y$

Rule *:
0 pts for problem if more than 1 item is wrong. Limits count as 1 item in this rule

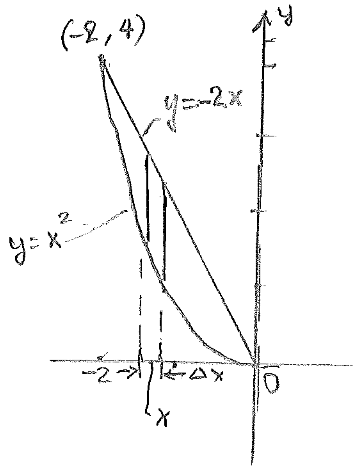
$\boxed{A = \int_{-1}^{1} \underbrace{[e^y - (y^2-2)]}_{} \, dy}$

(1) (5) (1) (1)

8

(8) 7. Let R be the region bounded by the curves $y = x^2$ and $y = -2x$. Use the method of washers to set up an integral for the volume V of the solid obtained by rotating R about the x–axis. Do not evaluate the integral.

Points of intersection: $x^2 = -2x \rightarrow x(x+2) = 0 \rightarrow x = -2, 0$



Volume of typical approximating washer:

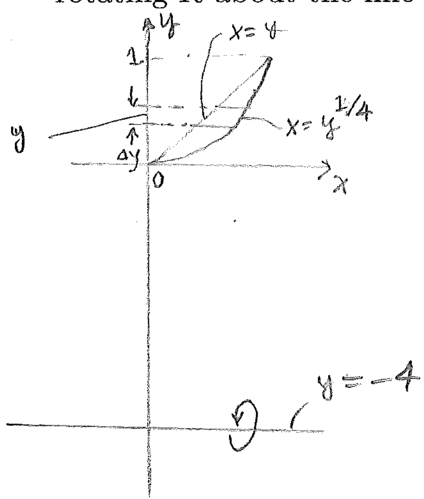
$$\Delta V = \left[\pi(-2x)^2 - \pi(x^2)^2\right]\Delta x$$

Rule *

$$\boxed{V = \int_{-2}^{0} \left(\pi 4x^2 - \pi x^4\right) dx}$$

8

(8) 8. Let R be the region bounded by the curves $y = x^4$ and $y = x$. Use the method of cylindrical shells to set up an integral for the volume V of the solid obtained by rotating R about the line $y = -4$. Do not evaluate the integral.



Volume of typical approximating cylindrical shell:

$$\Delta V = 2\pi[y - (-4)]\,[y^{1/4} - y]\,\Delta y$$

Rule *

$$\boxed{V = \int_{0}^{1} 2\pi(y+4)(y^{1/4} - y)\, dy}$$

8

(8) 9. A force of 10 lb is required to hold a spring stretched 4 in. beyond its natural length. How much work is done in stretching it from its natural length to 6 in. beyond its natural length?

$F = kx \quad F = 10\text{lb} \quad x = \frac{1}{3}ft \quad 10 = k\frac{1}{3} \rightarrow k = 30$ 4

Work: $W = \int_0^{\frac{1}{2}} 30x\,dx = 15x^2\Big|_0^{1/2} = 15 \cdot \frac{1}{4} = \frac{15}{4}$ ft-lbs

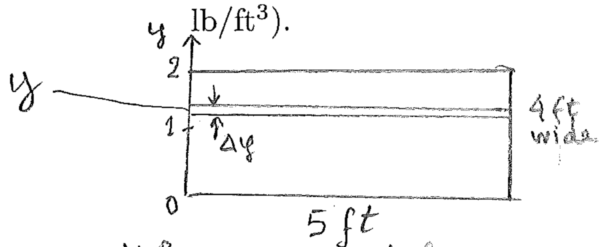
-3 pts if inches are not changed to feet, and the answers are correct using consistency

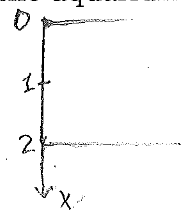
4

$$\boxed{W = \frac{15}{4} \text{ ft-lbs}}$$

8

(11) 10. An aquarium 5 ft long, 4 ft wide and 2 ft deep is full of water. Find the work needed to pump half of the water out of the aquarium. (Use the fact that water weighs 62.5 lb/ft^3).





Or (using the coordinate choice in Ex. 5 p 440)

$$W = \int_0^1 \underbrace{x(62.5)20}_{5}\, \underbrace{dx}_{1} \quad \text{(Rule *)}$$

1 1

Volume of typical layer $20\Delta y$

Weight " " " $(62.5)20\Delta y$

Work needed for lifting typical layer from y to 2: $\Delta W = (2-y)(62.5)20\Delta y$

$$W = \int_1^2 \underbrace{(2-y)(62.5)20}_{5}\,\underbrace{dy}_{1} \quad \text{(Rule *)}$$

1

$$\int_1^2 (2-y)dy = \left(2y - \frac{y^2}{2}\right)\Big|_1^2 = (4-2)-\left(2-\frac{1}{2}\right) = \frac{1}{2}$$

3

$\boxed{625}$ ft-lbs

11

(8) 11. $\int \sin^{-1} x\, dx =$

Integration by parts

$u = \sin^{-1} x \quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}} dx \quad v = x$

$$= x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx = x\sin^{-1}x + \sqrt{1-x^2} + C$$

$$\int \frac{x}{\sqrt{1-x^2}} dx = -\frac{1}{2}\int \frac{1}{z^{1/2}} dz = -\frac{1}{2}\frac{z^{1/2}}{\frac{1}{2}} = -\sqrt{1-x^2}$$

$z = 1-x^2$

$dz = -2x\,dx$

3 4 1

$\boxed{x\sin^{-1}x + \sqrt{1-x^2} + C}$

8

(11) 12. First make a substitution $x =$? and then use integration by parts to evaluate the integral $\int \theta^3 \cos(\theta^2) d\theta$. Your answer in the box must be in terms of θ.

$$\int \theta^3 \cos(\theta^2) d\theta = \frac{1}{2}\int x\cos x\, dx = \frac{1}{2}\left[x\sin x + \cos x\right] + C =$$

$x = \theta^2 \quad dx = 2\theta\, d\theta$

$$\int x\cos x\, dx = x\sin x - \int \sin x\, dx = x\sin x + \cos x$$

$u = x \quad dv = \cos x\, dx$

$du = dx \quad v = \sin x$

$$= \frac{1}{2}\theta^2 \sin(\theta)^2 + \frac{1}{2}\cos(\theta)^2 + C$$

-2 pts if $\frac{1}{2}$ is missing in one or both terms

5 5 1

$\boxed{\frac{1}{2}\theta^2\sin(\theta^2) + \frac{1}{2}\cos(\theta^2) + C}$

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