1. Find the integral by means of the substitution \( u = \sqrt{x} \).

\[
\int \frac{\sqrt{x} + 1}{x + 1} \, dx = \int \frac{u + 1}{u^2 + 1} \, 2u \, du \tag{4}
\]

\[
u = \sqrt{x} \quad x = u^2 \quad dx = 2u \, du
\]

\[
= 2 \int \frac{u^2 + u}{u^2 + 1} \, du = 2 \int \left(1 + \frac{u - 1}{u^2 + 1}\right) \, du \tag{2}
\]

\[
= 2 \int \left[1 + \frac{u}{u^2 + 1} - \frac{1}{u^2 + 1}\right] \, du
\]

\[
= 2 \left[u + \frac{1}{2} \ln(u^2 + 1) - \tan^{-1}u\right] + C
\]

\[
= 2\sqrt{x} + \ln(x + 1) - 2 \tan^{-1}\sqrt{x} + C
\]

-1 pt for missing \( dx \) or \( du \) (one time only for this problem)
-1 pt for missing \( + C \) (one time only for test)
2. Find the integral \( \int \frac{3x}{x^2 - 4x + 4} \, dx \).

\[
\frac{3x}{x^2 - 4x + 4} = \frac{3x}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2} \]

\[
3 = A \quad \quad 0 = -2A + B \]

\[A = 3, \quad B = 6\]

\[
\int \frac{3x}{x^2 - 4x + 4} \, dx = \int \frac{3}{x-2} \, dx + \int \frac{6}{(x-2)^2} \, dx
\]

\[= 3 \ln |x-2| - \frac{6}{x-2} + C\]

3. Determine whether the improper integral converges or diverges. If it converges find its value. Important: Show clearly how limits are involved.

(a) \( \int_1^\infty \frac{1}{x^{3/2}} \, dx \)

\[= \lim_{b \to \infty} \left[ -\frac{2}{\sqrt{x}} \right]_1^b \]

\[= \lim_{b \to \infty} \left[ -\frac{2}{\sqrt{b}} + 2 \right] = 2\]

(b) \( \int_0^\pi \tan x \, dx \)

\[= \lim_{b \to \frac{\pi}{2}} \left[ -\ln |\cos x| \right]_0^b \]

\[= \lim_{b \to \frac{\pi}{2}} \left[ -\ln (\cos b) + \ln 1 \right] = +\infty\]

\[\text{(diverges)}\]
(10) 4. Let \( R \) be the region between the graphs of the equations \( y = 3x \) and \( y = x^2 + 2 \). Use the washer method to set up an integral for the volume \( V \) of the solid generated by revolving the region \( R \) about the \( x \)-axis. Do not evaluate the integral.

\[
\begin{align*}
\frac{y}{3x} &= \frac{y}{x} + 2 \quad \rightarrow \quad x^2 + 2 = 3x \\
\frac{x^2 - 3x + 2}{(x-1)(x-2)} &= 0 \\
x &= 1, \quad x = 2 \\
\Delta V &= \pi \left[ (3x)^2 - (x^2 + 2)^2 \right] dx \\
V &= \int_{1}^{2} \pi \left[ 9x^2 - x^4 - 4x^2 - 4 \right] dx \\
&= 4 \pi \left[ 5x^2 - x^4 - 4 \right] dx
\end{align*}
\]

\[\text{x} \text{ or set for algebra mistake here}\]

(12) 5. Let \( R \) be the region between the graph of \( f(x) = \sin x \) and the \( x \)-axis on the interval \([\frac{\pi}{4}, \frac{\pi}{2}]\). Find the volume of the solid generated by revolving \( R \) about the \( y \)-axis.

\[
\begin{align*}
\Delta V &= 2 \pi x \sin x \Delta x \\
V &= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2 \pi x \sin x \, dx \\
&= 2 \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x \sin x \, dx = 2 \pi \left[ -x \cos x \left|_{\frac{\pi}{4}}^{\frac{\pi}{2}} \right. + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x \, dx \right] \\
&= 2 \pi \left[ -x \cos x + \sin x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} \\
&= 2 \pi \left[ 1 - \left( -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) \right] \pi \text{ or} \\
&= 2 \pi \left[ 1 - \frac{\pi}{2} \left( 1 - \frac{\pi}{4} \right) \right] \text{ or} \\
&= \pi \left[ 2 + \frac{\pi \sqrt{2}}{4} - \sqrt{2} \right]
\end{align*}
\]
6. Find the area $S$ of the surface generated by revolving about the $x$ axis the graph of $y = \frac{x^3}{3}$ on the interval $[0, 1]$.

\[
\Delta S = 2\pi \frac{x^3}{3} \sqrt{1 + (x^2)^2} \Delta x
\]

\[
S = \int_0^1 2\pi \frac{x^3}{3} \sqrt{1 + x^4} \, dx
\]

\[
\int x^3 \sqrt{1 + x^4} \, dx = \frac{1}{4} \int u^{3/2} \, du = \frac{1}{4} \frac{u^{3/2}}{3/2} = \frac{1}{6} (1 + x^4)^{3/2}
\]

\[
S = \frac{2\pi}{3} \frac{1}{6} (1 + x^4)^{3/2} \bigg|_0^1 = \frac{\pi}{9} (2\sqrt{2} - 1)
\]

\[
S = \frac{\pi}{9} (2\sqrt{2} - 1)
\]

7. A tank in the shape of an inverted cone 12 feet tall and 3 feet in radius is full of water. Set up an integral for the work $W$ required to pump all the water over the edge of the tank. Do not evaluate the integral. (Water weighs 62.5 lbs/ft$^3$).

\[
\frac{r'}{3} = \frac{x}{12} \quad \frac{r}{4} = \frac{x}{4}
\]

\[
\Delta W = (62.5) (12 - x) \frac{x}{4} \frac{x^2}{16} \Delta x
\]

\[
W = \int_0^{12} (62.5) (12 - x) \frac{x^2}{16} \, dx
\]

0 credit for problem if more than 2 items are wrong (limit count as 1 item in this rule).
(10) 8. Find the center of gravity \( (\bar{x}, \bar{y}) \) of the region \( R \) between the graphs of \( y = 2 - x \) and \( y = x - 2 \) on the interval \([0, 2]\). You may use symmetry for one of the coordinates, but must show complete work for the other.

From symmetry \( \bar{y} = 0 \)  
\[ \bar{x} \quad A = M_y \]
\[ A = \frac{1}{2} \cdot 4 \cdot 2 = 4 \]
\[ \Delta M_y = x \int_0^2 [(2-x) - (x-2)] \, dx \]
\[ M_y = \int_0^2 x (4 - 2x) \, dx \]
\[ = \left[ 4 \frac{x^2}{2} - 2 \frac{x^3}{3} \right]_0^2 = 4 \cdot \frac{4}{2} - 2 \cdot \frac{8}{3} = \frac{8}{3} \]
\[ \bar{x} = \frac{M_y}{A} = \frac{\frac{8}{3}}{4} = \frac{2}{3} \]

\( x = \frac{2}{3} \quad \bar{y} = 0 \)

(8) 9. Find the third Taylor polynomial \( p_3(x) \) of \( f(x) = \tan^{-1} x \).

\[ p_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 \]
\[ f(x) = \tan^{-1} x \quad f(0) = 0 \]
\[ f'(x) = \frac{1}{1+x^2} \quad f'(0) = 1 \]
\[ f''(x) = \frac{-2x}{(1+x^2)^2} \quad f''(0) = 0 \]
\[ f'''(x) = \frac{(1+x^2)^2(-2) + 2x \cdot 2(1+x^2)2x}{(1+x^2)^4} \quad f'''(0) = -2 \]

-2 pts for higher order terms
or \( p_n(x) \).

\[ p_3(x) = x - \frac{x^3}{3} \]