NAME SOLUTIONS

10-DIGIT PUID # ____________

RECITATION INSTRUCTOR ____________

RECITATION TIME ____________

LECTURER ____________

INSTRUCTIONS

1. There are 11 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. Also write your name at the top of pages 2–11.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet. No partial credit will be given, but if you show your work on the test booklet, it may be used in borderline cases.
4. No books, notes, calculators, or any electronic devices may be used on this exam.
5. Each problem is worth 8 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your 10-digit PUID, and fill in the little circles.
   (d) Using a #2 pencil, put your answers to questions 1–25 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your recitation instructor.

1. If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$ and $\vec{b} = -\vec{i} + \vec{j} + \vec{k}$, find a unit vector orthogonal to both $\vec{a}$ and $\vec{b}$ and having negative $\vec{k}$-component.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = -2\vec{j} + 2\vec{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{4+4} = 2\sqrt{2}$$

unit vector $\perp$ to both $\vec{a}$ and $\vec{b}$:

$$-\frac{1}{\sqrt{2}}\vec{j} + \frac{1}{\sqrt{2}}\vec{k}$$

and with negative $\vec{k}$-component:

$$\frac{1}{\sqrt{2}}\vec{j} - \frac{1}{\sqrt{2}}\vec{k}$$

A. $\vec{j} - \vec{k}$

B. $-\frac{1}{\sqrt{2}}\vec{j} - \frac{1}{\sqrt{2}}\vec{k}$

* C. $\frac{1}{\sqrt{2}}\vec{j} - \frac{1}{\sqrt{2}}\vec{k}$

D. $\frac{1}{\sqrt{3}}\vec{i} - \frac{\sqrt{2}}{\sqrt{3}}\vec{k}$

E. $\sqrt{\frac{2}{3}}\vec{j} - \frac{1}{\sqrt{3}}\vec{k}$

2. The vertices of a triangle are $P = (-1, 0, 1)$, $Q = (1, 1, 3)$ and $R = (2, 1, 0)$. If θ is the angle of the triangle at P, then $\cos\theta =$

$$\overrightarrow{PQ} = 2\vec{i} + \vec{j} + 2\vec{k}$$

$$\overrightarrow{PR} = 3\vec{i} + \vec{j} - \vec{k}$$

$$\overrightarrow{PQ} \cdot \overrightarrow{PR} = |\overrightarrow{PQ}|\,|\overrightarrow{PR}| \cos\theta$$

$$6 + 1 - 2 = \sqrt{9}\sqrt{11} \cos\theta$$

$$\cos\theta = \frac{5}{3\sqrt{11}}$$

A. $\frac{5}{3\sqrt{10}}$

B. $\frac{1}{3\sqrt{10}}$

C. 0

* D. $\frac{5}{3\sqrt{11}}$

E. $\frac{2}{3\sqrt{11}}$

3. The radius of the sphere $x^2 + y^2 + z^2 + 2x - 4z = 3$ is

$$x^2 + 2x + y^2 + z^2 - 4z = 3$$

$$x^2 + 2x + 1 + y^2 + z^2 - 4z + 4 = 3 + 1 + 4$$

$$(x+1)^2 + y^2 + (z-2)^2 = 8$$

$$\text{Radius} = \sqrt{8} = 2\sqrt{2}$$

* A. $2\sqrt{2}$

B. $\sqrt{2}$

C. $\sqrt{11}$

D. $\sqrt{17}$

E. 1

4. $\lim_{n\to\infty} \frac{\ln(1+n^2)}{\ln(1+n)} = \lim_{x\to\infty} \frac{\ln(1+x^2)}{\ln(1+x)} =$

$\frac{\infty}{\infty}$

$\overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{2x}{1+x^2}}{\frac{1}{1+x}} = \lim_{x\to\infty} \frac{2x+2x^2}{1+x^2} = 2$

A. 0

B. $+\infty$

C. 1

* D. 2

E. $\frac{1}{2}$

5. $\int_1^\infty \frac{1}{1+x^2}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{1+x^2}\,dx$

$= \lim_{t\to\infty} \left[\tan^{-1}x\right]_1^t$

$= \lim_{t\to\infty} \left[\tan^{-1}t - \tan^{-1}1\right]$

$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

A. $\frac{\pi}{3}$

B. 1

C. $-\frac{\pi}{4}$

D. $\frac{\pi}{2}$

* E. $\frac{\pi}{4}$

6. $\int_0^{\frac{\pi}{2}} (\sin x + \cos x)^2 dx = \int_0^{\frac{\pi}{2}} (\sin^2 x + \cos^2 x + 2\sin x\cos x)\,dx$

$= \int_0^{\frac{\pi}{2}} (1 + 2\sin x\cos x)\,dx$

$= \left[x + \sin^2 x\right]_0^{\frac{\pi}{2}}$

$= \frac{\pi}{2} + 1$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{2}+\frac{1}{2}$

C. 1

D. $\frac{\pi}{2}-\frac{1}{2}$

* E. $\frac{\pi}{2}+1$

7. $\int_2^3 \frac{1}{x^2 - x}\,dx =$

$$\frac{1}{x^2-x} = \frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$$1 = Ax - A + Bx$$

$$\left.\begin{matrix} A+B=0 \\ -A = 1 \end{matrix}\right] \to A=-1,\ B=1$$

$$\int_2^3 \frac{1}{x^2-x}\,dx = \int_2^3 \left(-\frac{1}{x} + \frac{1}{x-1}\right)dx$$

$$= \Big[-\ln x + \ln(x-1)\Big]_2^3 = -\ln 3 + \ln 2 - (-\ln 2 + \ln 1)$$

$$= -\ln 3 + 2\ln 2 = \ln\frac{4}{3}$$

A. $\ln\frac{2}{3}$

* B. $\ln\frac{4}{3}$

C. $\ln\frac{1}{6}$

D. $\ln\frac{1}{3}$

E. $\ln\frac{3}{2}$

8. $\int x(\ln x)^4 dx =$

Integration by parts:

$u = (\ln x)^4$, $dv = x\,dx$ $\qquad \int u\,dv = uv - \int v\,du$

$du = 4(\ln x)^3 \frac{1}{x}dx$, $v = \frac{x^2}{2}$

$$\int x(\ln x)^4 dx = \frac{x^2}{2}(\ln x)^4 - \int \frac{x^2}{2} 4(\ln x)^3 \frac{1}{x}dx$$

$$= \frac{x^2}{2}(\ln x)^4 - 2\int x(\ln x)^3 dx$$

A. $(\ln x)^4 - 4\int x(\ln x)^3 dx$

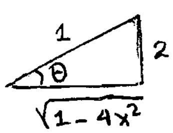
* B. $\frac{x^2}{2}(\ln x)^4 - 2\int x(\ln x)^3 dx$

C. $4x(\ln x)^3 - \int (\ln x)^3 dx$

D. $\frac{x^2}{10}(\ln x)^5 - \frac{1}{5}\int (\ln x)^5 dx$

E. $\frac{1}{5}(\ln x)^5 - \frac{1}{5}\int x(\ln x)^5 dx$

9. The trigonometric substitution $2x = \sin\theta$ converts the integral $\int_{\frac{1}{4}}^{\frac{1}{2}} (1-4x^2)^{\frac{5}{2}}\,dx$ into



$$2x = \sin\theta$$

$$dx = \frac{1}{2}\cos\theta\,d\theta$$

$$\sqrt{1-4x^2} = \cos\theta$$

$$x = \tfrac{1}{4} \to \theta = \tfrac{\pi}{6}\ ;\ x = \tfrac{1}{2} \to \theta = \tfrac{\pi}{2}$$

$$\int_{\frac{1}{4}}^{\frac{1}{2}} \left(\sqrt{1-4x^2}\right)^5 dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\cos^5\theta)\frac{1}{2}\cos\theta\,d\theta$$

$$= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^6\theta\,d\theta$$

A. $\frac{1}{2}\int_{\frac{1}{2}}^{1} \cos^5\theta\sin\theta\,d\theta$

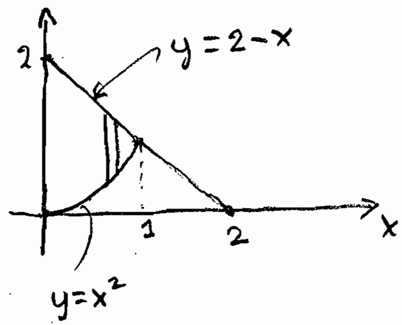
B. $64\int_{\frac{1}{2}}^{1} \cos^6\theta\,d\theta$

C. $-\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^5\theta\sin\theta\,d\theta$

* D. $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^6\theta\,d\theta$

E. $64\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^6\theta\,d\theta$

10. The region in the first quadrant bounded by the y-axis, the graph of $x + y = 2$ and the graph of $y = x^2$ is revolved about the x-axis. The volume V of the solid generated is



Point of intersection of $y = 2-x$ and $y = x^2$:

$x^2 = 2-x$

$x^2 + x - 2 = 0$, $(x+2)(x-1)=0$

$x = 1, -2$

A. $\frac{\pi}{2}$

B. $\frac{3\pi}{5}$

C. 4π

D. $\frac{14\pi}{3}$

* E. $\frac{32\pi}{15}$

Volume of typical washer

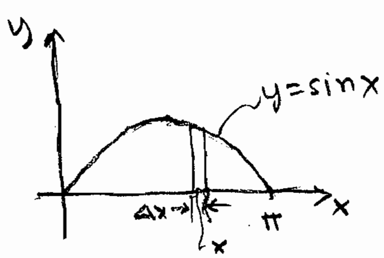
$$\Delta V = \left[\pi(2-x)^2 - \pi(x^2)^2\right]\Delta x$$

$$V = \int_0^1 \left[\pi(2-x)^2 - \pi x^4\right]dx = \pi\int_0^1 (4 - 4x + x^2 - x^4)\,dx$$

$$= \pi\left(4x - 2x^2 + \frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1$$

$$= \pi\left(4 - 2 + \frac{1}{3} - \frac{1}{5}\right) = \pi\,\frac{30+5-3}{15} = \pi\,\frac{32}{15}$$

11. The region bounded by the graph of $y = \sin x$, $0 \le x \le \pi$, and the x-axis, is revolved about the y-axis. The volume V of the solid generated is



Volume of typical shell:

$\Delta V = 2\pi x \sin x\, \Delta x$

A. $\frac{\pi}{2}$

B. 2π

* C. $2\pi^2$

D. π^2

E. π

$$V = \int_0^\pi 2\pi x \sin x\, dx = 2\pi\int_0^\pi x \sin x\, dx$$

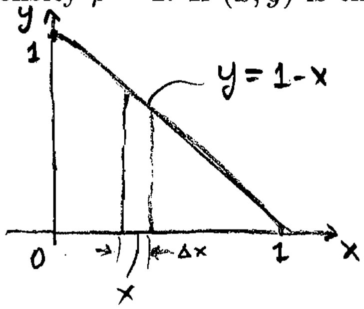
$u = x \quad dv = \sin x\, dx$

$du = dx \quad v = -\cos x$

$$= 2\pi\left[-x\cos x\Big|_0^\pi + \int_0^\pi \cos x\, dx\right]$$

$$= 2\pi\left[-\pi\cos\pi + (\sin x)\Big|_0^\pi\right] = 2\pi^2$$

12. Consider the lamina bounded by the curves $x + y = 1$, $x = 0$, and $y = 0$ and with density $\rho = 1$. If $(\overline{x}, \overline{y})$ is the center of mass of the lamina, then $\overline{x} =$



$$m\overline{x} = M_y$$
$$m = 1 \cdot \frac{1}{2} = \frac{1}{2}$$

$$M_y = \int_0^1 x \cdot 1(1-x)\,dx$$
$$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$
$$\frac{1}{2}\overline{x} = \frac{1}{6} \rightarrow \overline{x} = \frac{1}{3}$$

* A. $\frac{1}{3}$

B. $\frac{1}{4}$

C. $\frac{3}{8}$

D. $\frac{1}{2}$

E. $\frac{2}{5}$

13. Suppose that $\sum_{n=1}^{\infty} a_n = \frac{\pi}{2}$. Let $b_n = 2a_n$ and $s_n = b_1 + b_2 + \cdots + b_n$.
Which one of these statements is true?

* A. $\lim_{n\to\infty} s_n = \pi$ and $\lim_{n\to\infty} b_n = 0$

B. $\lim_{n\to\infty} s_n = \pi$ but $\lim_{n\to\infty} b_n$ cannot be determined

C. $\lim_{n\to\infty} s_n = \infty$ and $\lim_{n\to\infty} b_n = \pi$

D. $\lim_{n\to\infty} b_n = 0$ but $\lim_{n\to\infty} s_n$ cannot be determined

E. $\lim_{n\to\infty} s_n = 0$ but $\lim_{n\to\infty} b_n$ cannot be determined

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} 2a_n = 2\sum_{n=1}^{\infty} a_n = 2\,\frac{\pi}{2} = \pi$$

By definition; $\lim_{n\to\infty} s_n = \sum_{n=1}^{\infty} b_n = \pi$

and since $\sum_{n=1}^{\infty} b_n$ is convergent, $\lim_{n\to\infty} b_n = 0$

14. The series $\sum_{n=1}^{\infty} \frac{1}{e^n}$ $= \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \cdots$

$= \frac{1}{e}\left[1 + \frac{1}{e} + \left(\frac{1}{e}\right)^2 + \left(\frac{1}{e}\right)^3 + \cdots\right]$

$= \frac{1}{e} \frac{1}{1 - \frac{1}{e}} = \frac{1}{e-1}$

A. diverges

*B. $= \frac{1}{e-1}$

C. $= \frac{e}{e-1}$

D. $= \frac{1}{1-e}$

E. $= \frac{1}{1+e}$

15. The series $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n^2}\right)$

$\lim_{n\to\infty} \left(1 + \frac{1}{n^2}\right) = 1 \neq 0$

$\therefore \sum_{n=1}^{\infty} \left(1 + \frac{1}{n^2}\right)$ diverges

A. Converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

B. Converges by the ratio test

C. Diverges by the ratio test

D. Converges by the limit comparison test

* E. Diverges

16. Which of these series converge?

(I) $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n^2+5}}$, Limit comp. test compare with $\sum_{n=1}^{\infty} \frac{1}{n}$ div.

(II) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{\sqrt{n^3+5}}$, Comparison test compare with $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ conv.

(III) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^5+2n^2+1}}$ Comparison test compare with $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ conv.

A. All

B. Only (III)

C. Only (II)

D. Only (I) and (III)

* E. Only (II) and (III)

17. Of the series

(I) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$, (II) $\sum_{n=1}^{\infty} (-1)^n \left(1+\frac{1}{n}\right)$, (III) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n^3-2}}$

(I) conv. by Alt. ser. test but does not conv. abs. compare $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ with $\sum_{n=2}^{\infty} \frac{1}{n}$

(II) $\lim_{n\to\infty} (-1)^n(1+\frac{1}{n})$ DNE $\therefore$ series div.

(III) conv. abs. compare $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^3-2}}$ with $\sum_{n=2}^{\infty} \frac{1}{n^{3/2}}$ (lim. comp. test)

A. (I) and (III) converge absolutely

B. (I) and (II) converge absolutely

* C. Only (III) converges absolutely

D. All converge but none converges absolutely

E. (I) and (II) diverge

18. If $f(x) = \tan x$, the terms of the Maclaurin series of f up to the third power of x are

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + \frac{f^{(1)}(0)}{1!} + \frac{f^{(2)}(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \cdots$$

$f(x) = \tan x$ $\quad f(0) = 0$

$f^{(1)}(x) = \sec^2 x$ $\quad f^{(1)}(0) = 1$

$f^{(2)}(x) = 2\sec x \sec x \tan x = 2\sec^2 x \tan x$ $\quad f^{(2)}(0) = 0$

$f^{(3)}(x) = 2\sec^2 x \sec^2 x + 4\sec x \sec x \tan x \tan x$

$f^{(3)}(0) = 2$

$\tan x = x + \frac{2}{3!}x^3 + \cdots = x + \frac{1}{3}x^3 + \cdots$

A. $x - x^2 - \frac{x^3}{3}$

* B. $x + \frac{x^3}{3}$

C. $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

D. $x + \frac{x^3}{3!}$

E. $\frac{x}{1+x^2} - \frac{x^3}{(1+x^2)^2}$

19. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n^2}{(n+1)!} x^n$ is

* A. ∞
B. 1
C. 2
D. e
E. 0

Ratio test

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{(n+1)^2}{(n+2)!}x^{n+1}}{\frac{n^2}{(n+1)!}x^n}\right|$$

$$= \frac{(n+1)^2(n+1)!}{n^2(n+2)!}|x|.$$

$$= \left(\frac{n+1}{n}\right)^2 \frac{1}{n+2}|x| \xrightarrow[\text{as } n\to\infty]{} 1\cdot 0\cdot |x| = 0 < 1$$

$\therefore$ series conv. for all x and $R = \infty$

20. Match the functions with their Maclaurin series.

(1)	e^x	(a)	$\sum_{n=0}^{\infty} (-1)^n x^n, \ -1 < x < 1$
(2)	$\frac{1}{1+x}$	(b)	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, \ -\infty < x < \infty$
(3)	$\frac{x}{1-x}$	(c)	$x + x^2 + x^3 + \ldots, \ -1 < x < 1$
(4)	$x\sin x$	(d)	$1 - \frac{3^2x^2}{2!} + \frac{3^4x^4}{4!} - \ldots, \ -\infty < x < \infty$
(5)	$\cos 3x$	(e)	$x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \ldots, \ -\infty < x < \infty$

A. 1b,2c,3a,4d,5e
B. 1e,2a,3c,4b,5d
C. 1a,2b,3d,4c,5e
D. 1b,2c,3a,4e,5d
* E. 1b,2a,3c,4e,5d

(1) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ (b)

(2) $\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty}(-x)^n = \sum_{n=0}^{\infty}(-1)^n x^n$ (a)

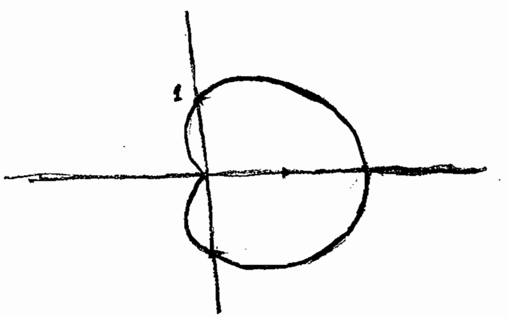
(3) $\frac{x}{1-x} = x\sum_{n=0}^{\infty} x^n = x(1+x+x^2+\cdots)$
$= x + x^2 + x^3 + \cdots$ (c)

(4) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$
$x\sin x = x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \cdots$ (e)

(5) $\cos 3x = 1 - \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!} - \cdots = 1 - \frac{3^2x^2}{2!} + \frac{3^4x^4}{4!} - \cdots$ (d)

21. The graph of the polar equation $r = 1 + \cos\theta$ is

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
r	2	1	0	1	2



A. A circle with center at $(x, y) = (0, 1)$

B. A circle with center at $(x, y) = (1, 0)$

C. A two-leaved rose

* D. A cardioid with the point farthest from the origin at $(x, y) = (2, 0)$

E. A cardioid with the point farthest from the origin at $(x, y) = (0, 2)$

22. Convert the polar equation $r = -2\cos\theta$ to rectangular coordinates

$$r^2 = -2r\cos\theta$$
$$x^2 + y^2 = -2x$$
$$x^2 + 2x \quad + y^2 = 0$$
$$x^2 + 2x + 1 + y^2 = 1$$
$$(x+1)^2 + y^2 = 1$$

A. $(x - 1)^2 + y^2 = 1$

* B. $(x + 1)^2 + y^2 = 1$

C. $x^2 + (y - 1)^2 = 1$

D. $x^2 + (y + 1)^2 = 1$

E. $x^2 + y^2 = 2$

23. The curve described parametrically by

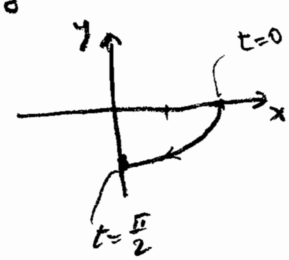
$$x = 2\cos t, \qquad y = -\sin t; \qquad 0 \le t \le \frac{\pi}{2}$$

is

$$\frac{x^2}{2^2} + \frac{y^2}{1^2} = 1 \qquad x > 0, \quad y < 0$$

$t = 0$: $(x, y) = (2, 0)$

$t = \frac{\pi}{2}$: $(x, y) = (0, -1)$



A. an ellipse

B. a quarter of a circle

C. a half of a circle

D. a half of an ellipse

* E. a quarter of an ellipse

24. The length L of the curve in problem 23 is given by

$$L = \int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{(-2\sin t)^2 + (-\cos t)^2}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{4\sin^2 t + \cos^2 t}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{3\sin^2 t + 1}\, dt$$

* A. $\int_0^{\frac{\pi}{2}} \sqrt{3\sin^2 t + 1}\, dt$

B. $\int_0^{\frac{\pi}{2}} \sqrt{2\cos^2 t + 1}\, dt$

C. $\int_0^{\frac{\pi}{2}} \sqrt{-\sin^2 t + 1} dt$

D. $\int_0^{\frac{\pi}{2}} (-2\sin t + \cos t) dt$

E. $\int_0^{\frac{\pi}{2}} (\sqrt{3}\sin t + 1) dt$

25. The polar form of the complex number $\dfrac{i}{1+\sqrt{3}\,i}$ with argument between 0 and 2π is

$$i = 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$1+\sqrt{3}\,i = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$\frac{i}{1+\sqrt{3}\,i} = \frac{1}{2}\left[\cos\left(\frac{\pi}{2}-\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{2}-\frac{\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

or

$$\frac{i}{1+\sqrt{3}\,i} = \frac{i}{1+\sqrt{3}i}\cdot\frac{1-\sqrt{3}i}{1-\sqrt{3}i} = \frac{\sqrt{3}+i}{4} = \frac{\sqrt{3}}{4} + \frac{1}{4}i$$

$$r^2 = \frac{3}{16} + \frac{1}{16} = \frac{4}{16} = \frac{1}{4} \qquad r = \frac{1}{2}$$

$$\tan\theta = \frac{\frac{1}{4}}{\frac{\sqrt{3}}{4}} = \frac{1}{\sqrt{3}} \qquad \theta = \frac{\pi}{6}$$

$$\frac{i}{1+\sqrt{3}i} = \frac{1}{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

A. $\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

B. $\frac{1}{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

C. $\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

* D. $\frac{1}{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

E. $\sqrt{3}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$