NAME Solutions

STUDENT ID # __________

INSTRUCTOR __________

INSTRUCTIONS

1. There are 6 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. I.D.# is your 9 digit ID (probably your social security number). Also write your name at the top of pages 2–6.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet for the first 7 questions. Partial credit will be given for work on the last 3 questions.
4. No books, notes or calculators may be used on this exam.
5. Each problem is worth 10 points. The maximum possible score is 100 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your student ID number, and fill in the little circles.
   (d) Using a #2 pencil, put your answers to questions 1–7 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
   (e) Sign your answer sheet.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your instructor.

1. The set of all points whose coordinates satisfy the equation

$$x^2 + y^2 + z^2 + 6x + 8y - 4z + 4 = 0$$

is

A. a sphere with center $(3, 4, -2)$ and radius 25

B. a sphere with center $(3, 4, -2)$ and radius 5

C. a sphere with center $(-3, -4, 2)$ and radius $\sqrt{33}$

D. a sphere with center $(-3, -4, 2)$ and radius 5

E. None of the above

Complete squares

$x^2+6x+9+y^2+8y+z^2-4z+4 = 9+16+4-4$

$(x+3)^2+(y+4)^2+(z-2)^2 = 25$

sphere with center at $(-3,-4,2)$ and radius 5

2. Find parametric equations of the line that is perpendicular to the plane

$$\mathcal{P}: \qquad 2x + 3y - z = 8$$

and passes through the point of intersection of $\mathcal{P}$ with the x-axis.

A. $x = 4 + 2t,\ y = 3t,\ z = -t$

B. $x = 2 + 4t,\ y = 3 + t,\ z = -1 - t$

C. $x = 4 - 2t,\ y = -3t,\ z = -t$

D. $x = 2t,\ y = 3t.\ z = -t$

E. $x = 4 + 2t,\ y = \frac{8}{3} + 3t,\ z = -8 - t$

Find first the intersection of $\mathcal{P}$ with the x-axis:

$y=z=0 \quad 2x + 3\cdot 0 - 0 = 8$

$x = 4$

$(x_0, y_0, z_0) = (4, 0, 0)$

Normal to plane $\mathcal{P}$: $\vec{N} = 2\vec{i} + 3\vec{j} - \vec{k}$

Direction vector of line $\vec{L} = \vec{N} = 2\vec{i} + 3\vec{j} - \vec{k}$

$x = 4 + 2t \quad y = 3t \quad z = -t$

3. The curve traced out by the vector-valued function

$$\vec{F}(t) = t\vec{j} + (1-t^2)\vec{k}, \quad -1 \le t \le 1$$

is part of

Parametric equations of curve:
$x = 0$, $y = t$, $z = 1 - t^2$

$x = 0$: yz-plane

$z = 1 - y^2$

parabola

starting at $(0,-1,0)$ when $t=-1$

ending at $(0,1,0)$ when $t=1$

A. a circle in the xz-plane starting at $(0,-1,0)$ and ending at $(0,1,0)$

B. a parabola in the yz-plane starting at $(0,-1,0)$ and ending at $(0,1,0)$

C. an ellipse in the yz-plane going through the point $(0,0,1)$

D. a circle in the yz-plane going through the point $(0,0,1)$

E. a hyperbola in the yz-plane starting at $(0,-1,0)$ and ending at $(0,1,0)$

4. Compute the length of the curve

$$\vec{r}(t) = \frac{t^3}{3}\vec{i} + t^2\vec{j} + \vec{k}, \quad 0 \le t \le \sqrt{5}$$

$$L = \int_0^{\sqrt{5}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

$$= \int_0^{\sqrt{5}} \sqrt{(t^2)^2 + (2t)^2 + 0^2}\, dt$$

$$= \int_0^{\sqrt{5}} \sqrt{t^4 + 4t^2}\, dt$$

$$= \int_0^{\sqrt{5}} \sqrt{t^2 + 4}\; t\, dt$$

$u = t^2 + 4$
$du = 2t\, dt$

$$= \frac{1}{2} \frac{(t^2+4)^{3/2}}{\frac{3}{2}} \Bigg|_0^{\sqrt{5}}$$

$$= \frac{(5+4)^{3/2}}{3} - \frac{4^{3/2}}{3} = \frac{27}{3} - \frac{8}{3} = \frac{19}{3}$$

A. $\frac{5^{3/2} - 1}{3}$

B. $\frac{5^{3/2} - 8}{3}$

C. 3

D. $5^{3/2}$

E. $\frac{19}{3}$

5. Find the unit tangent vector to the curve $\vec{r}(t) = 2t^2\vec{i} + t^3\vec{j} + \vec{k}$ at the point $(2, 1, 1)$.

$2\vec{i} + \vec{j} + \vec{k} = 2t^2\vec{i} + t^3\vec{j} + \vec{k}$

$$\left.\begin{array}{l} 2 = 2t^2 \\ 1 = t^3 \\ 1 = 1 \end{array}\right] \Rightarrow t = 1$$

$\vec{r}\,'(t) = 4t\vec{i} + 3t^2\vec{j}$

$\vec{r}\,'(1) = 4\vec{i} + 3\vec{j}$

$\|\vec{r}\,'(1)\| = \sqrt{16+9} = 5$

$\vec{u} = \dfrac{\vec{r}\,'(1)}{\|\vec{r}\,'(1)\|} = \frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}$

A. $\frac{4}{5}\vec{i} - \frac{3}{5}j$

B. $3\vec{i} + 4j$

C. $\frac{4}{5}i + \frac{3}{5}j$

D. $4\vec{i} + 3\vec{j}$

E. $\frac{2}{5}\vec{i} + \frac{3}{5}j + \frac{1}{5}\vec{k}$

6. The level surfaces of the function $f(x, y, z) = x^2 + 3y^2 - 5z$ are

$x^2 + 3y^2 - 5z = c$

$5z = x^2 + 3y^2 - c$

A. ellipsoids

B. paraboloids

C. cones

D. spheres

E. cylinders

7. Compute $\frac{\partial^2 f}{\partial x \partial y}$ if $f(x, y) = x^2 \sin xy$.

A. $2x \cos xy - x^2 y \sin xy$

B. $3x^2 \cos xy - x^3 y \sin xy$

C. $3x^2 \cos xy$

D. $2x \cos xy$

E. $2x \cos xy - x^2 \sin xy$

$$f(x, y) = x^2 \sin xy$$

$$\frac{\partial f}{\partial y} = x^2 (\cos xy) \cdot x$$

$$= x^3 \cos xy$$

$$\frac{\partial^2 f}{\partial x \partial y} = x^3 (-\sin xy) \cdot y + 3x^2 \cos xy$$

$$= -x^3 y \sin xy + 3x^2 \cos xy$$

8. Find an equation of the plane containing the points $\underset{A}{(2, 3, 1)}$, $\underset{B}{(1, 1, 5)}$ and $\underset{C}{(0, 1, 1)}$.

$$\vec{AB} = -\vec{i} - 2\vec{j} + 4\vec{k}$$

$$\vec{AC} = -2\vec{i} - 2\vec{j}$$

$$\vec{N} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -2 & 4 \\ -2 & -2 & 0 \end{vmatrix} = 8\vec{i} - 8\vec{j} - 2\vec{k}$$

$$8(x-2) - 8(y-3) - 2(z-1) = 0$$

9. A particle has acceleration $\vec{a}(t) = 2\vec{\imath} + e^t\vec{\jmath}$. The initial position is $\vec{r}(0) = \vec{\imath} + \vec{k}$ and the initial velocity is $\vec{v}(0) = \vec{\jmath}$. Find the velocity $\vec{v}(t)$ and the position vector $\vec{r}(t)$.

$$\vec{a}(t) = 2\vec{i} + e^t\vec{j}$$

$$\vec{v}(t) = \int \vec{a}(t)\,dt = 2t\vec{i} + e^t\vec{j} + \vec{C}$$

$t=0$: $\vec{v}(0) = 2\cdot 0\,\vec{i} + e^0\vec{j} + \vec{C}$

$$\vec{j} = \vec{j} + \vec{C} \Rightarrow \vec{C} = \vec{0}$$

$$\vec{v}(t) = 2t\vec{i} + e^t\vec{j}$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt = t^2\vec{i} + e^t\vec{j} + \vec{D}$$

$t=0$: $\vec{r}(0) = 0^2\cdot\vec{i} + e^0\vec{j} + \vec{D}$

$$\vec{i} + \vec{k} = \vec{j} + \vec{D} \Rightarrow \vec{D} = \vec{i} - \vec{j} + \vec{k}$$

$$\vec{r}(t) = t^2\vec{i} + e^t\vec{j} + \vec{i} - \vec{j} + \vec{k}$$

$\vec{v}(t) = 2t\vec{i} + e^t\vec{j}$
$\vec{r}(t) = (t^2+1)\vec{i} + (e^t-1)\vec{j} + \vec{k}$

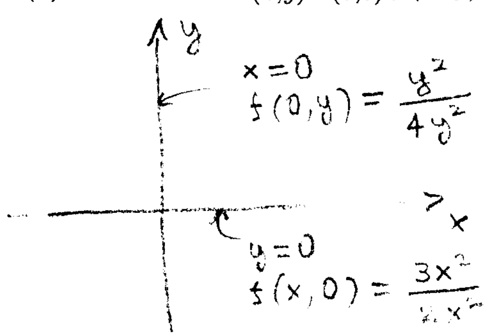
10. Let

$$f(x,y) = \frac{3x^2 + 2xy^2 + y^2}{2x^2 + 4y^2}$$

(a) What is the limit of $f(x,y)$ as (x,y) approaches $(0,0)$ along the x-axis.

(b) What is the limit of $f(x,y)$ as (x,y) approaches $(0,0)$ along the y-axis.

(c) Does the $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? If yes what is its value?



$$\lim_{x\to 0} f(x,0) = \lim_{x\to 0} \frac{3x^2}{2x^2} = \frac{3}{2}$$

$$\lim_{y\to 0} f(0,y) = \lim_{y\to 0} \frac{y^2}{4y^2} = \frac{1}{4}$$

$\therefore \lim_{(x,y)\to(0,0)} f(x,y)$ does not exist since the limits along

a. $\frac{3}{2}$
b. $\frac{1}{4}$
c. Does not exist