1. Find a vector function \( \mathbf{r}(t) \) that traces the line which contains the point \((3, 4, 0)\) and is perpendicular to the plane \( z = 2x - 5y + 7 \).

- A. \( \mathbf{r}(t) = (2 + 3t, -5 + 4t, 1) \)
- B. \( \mathbf{r}(t) = (3 - t, 2 + 4t, -t) \)
- C. \( \mathbf{r}(t) = (1 + t, 2 - 3t, 7 + t) \)
- D. \( \mathbf{r}(t) = (3 + t, 4 + 5t, t) \)
- E. \( \mathbf{r}(t) = (3 + 2t, 4 - 5t, -t) \)

The plane is \( 2x - 5y - 2z + 7 = 0 \)

The line is \( (3, 4, 0) + t(n_1, n_2, n_3) \)

And \((n_1, n_2, n_3) = (2, -5, -1)\)

from the equation of the plane.
2. The approximate change of \( z = \sqrt{1 + x + y^2} \) as \((x, y)\) changes from \((2, 1)\) to \((1.9, 1.2)\) is

\[
\frac{\partial z}{\partial x} \bigg|_{(2,1)} = \frac{1}{2 \sqrt{1 + x + y^2}} \bigg|_{x=2, y=1} = \frac{1}{4}
\]

\[
\frac{\partial z}{\partial y} \bigg|_{(2,1)} = \frac{2y}{2 \sqrt{1 + x + y^2}} \bigg|_{(2,1)} = \frac{1}{2}
\]

A. \( \frac{1}{10} \)
B. \( \frac{1}{\sqrt{10}} \)
C. \( \frac{3}{40} \)
D. \( -\frac{1}{40} \)
E. \( -\frac{1}{20} \)

Answer: \( \frac{1}{4} \cdot (1.9 - 2) + \frac{1}{2} \cdot (1, 2 - 1) = \)

\[-\frac{1}{4} + \frac{4}{4} = \frac{3}{4} \]
3. The length of the path traced out by \( r(t) = 2t^{3/2} \mathbf{i} + \cos 2t \mathbf{j} + \sin 2t \mathbf{k} \) over the interval \( 0 \leq t \leq 2 \) is

A. \( \int_0^2 \sqrt{4t^3 + 4} \, dt \)

B. \( \int_0^2 4t^3 + 4 \, dt \)

C. \( \int_0^2 \sqrt{9t + 4} \, dt \)

D. \( \int_0^2 9t + 4 \, dt \)

E. \( \int_0^2 \frac{1}{\sqrt{4t^3 + 4}} \, dt \)

\[
\int_0^2 \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2} \, dt
\]

\[
= \int_0^2 \sqrt{(3t^{1/2})^2 + (-2\sin 2t)^2 + (2\cos 2t)^2} \, dt
\]

\[
= \int_0^2 \sqrt{9t + 4} \, dt
\]
4. Suppose \( f(7, 8) = 5, f(7.1, 8) = 5.1, f(7, 8.2) = 5.4, \) and \( f(7.1, 8.2) = 5.5. \) The best estimates for \( f_x(7, 8) \) and \( f_y(7, 8) \) based on this data are

A. \( f_x(7, 8) = 2 \) and \( f_y(7, 8) = 1 \)
B. \( f_x(7, 8) = 2 \) and \( f_y(7, 8) = 2 \)
C. \( f_x(7, 8) = 1 \) and \( f_y(7, 8) = 1 \)
D. \( f_x(7, 8) = 1 \) and \( f_y(7, 8) = 2 \)
E. \( f_x(7, 8) = 3 \) and \( f_y(7, 8) = 1 \)

\[
\begin{align*}
\hat{f}_x(7, 8) & \approx \frac{f(7, 1, 8) - f(7, 8)}{0.1} = \frac{0.1}{0.1} = 1 \\
\hat{f}_y(7, 8) & \approx \frac{f(7, 8, 2) - f(7, 8)}{0.2} = \frac{0.2}{0.2} = 2
\end{align*}
\]
5. Find the equation of the tangent plane to \( z = e^{xy} \) at the point \((1,1,e)\)

A. \( z = ex + ey + 1 \)
B. \( z = x + y + e - 2 \)
C. \( z = ex + ey + e \)
D. \( z = ex + ey - e \)
E. \( z = x + y + 1 \)

\[
\frac{\partial z}{\partial x} \bigg|_{(1,1)} = ye^x \bigg|_{(1,1)} = e^1 = e
\]

\[
\frac{\partial z}{\partial y} \bigg|_{(1,1)} = xe^y \bigg|_{(1,1)} = 1 \cdot e^1 = e
\]

Equation

\[ p(x,y) = Ax + By + C = ex + ey + C \]

to find \( C \) plug in \( p(1,1) = e \)

so \( e \cdot 1 + e \cdot 1 + C = e \)

\( C = -e \)

Or \( z - z_0 = A(x-x_0) + B(y-y_0) \)

to get \( z = ex + ey - e \)
6. The vector projection of $4j$ onto $4i + 4j$, that is, $\text{proj}_{4i+4j} 4j$, equals

A. $i + j$
B. $2i + 2j$
C. $3i + 3j$
D. $4i + 4j$
E. $4j$

The projection is long and points in the direction of $4i + 4j$, i.e., of the unit vector $\frac{i + j}{\sqrt{2}}$.

So, answer is:

$\text{Length of } 4j \times \cos \theta \times \frac{i + j}{\sqrt{2}}$

$= (4) \frac{1}{\sqrt{2}} \times \frac{x^2 + y^2}{\sqrt{2}} = \frac{4}{2} (x + y)$

$\boxed{B}$
7. Find \( b \) and \( c \) so that \( \mathbf{v} = \langle 4, b, c \rangle \) is parallel to the planes \( x + y + z = 3 \) and \( 2x + z = 0 \).

A. \( b = -8, c = 4 \)
B. \( b = 8, c = 4 \)
C. \( b = 12, c = 4 \)
D. \( b = -4, c = -8 \)
E. \( b = 4, c = -8 \)

The easiest way is to note that \( \langle 4, b, c \rangle \) must be perpendicular to both \( \mathbf{i} + \mathbf{j} + \mathbf{k} \), the normal vector for the first plane, and \( 2\mathbf{i} + \mathbf{k} \), the normal vector for the second.

So, \[
\left( 4\mathbf{i} + b\mathbf{j} + c\mathbf{k} \right) \cdot \left( \mathbf{i} + \mathbf{j} + \mathbf{k} \right) = 0
\]
\[
\left( 4\mathbf{i} + b\mathbf{j} + c\mathbf{k} \right) \cdot \left( 2\mathbf{i} + \mathbf{k} \right) = 0
\]

\[
\begin{cases}
4 + b + c = 0 \\
4 \cdot 2 + c \cdot 1 = 0
\end{cases}
\]
works only for \( b = 4 \).

\[
\begin{cases}
4 + 4 + c = 0 \\
4 \cdot 2 + c \cdot 1 = 0
\end{cases}
\]

\( c = -8, \text{ and } \)

Alternative, the answer must be parallel to \( \left( \mathbf{i} + \mathbf{j} + \mathbf{k} \right) \times \left( 2\mathbf{i} + \mathbf{k} \right) \).
8. The graph of $x^2 - 2y^2 + 3z^2 - 4 = 0$ is

A. A hyperboloid of one sheet which does not intersect the $x$ axis
B. A hyperboloid of one sheet which does not intersect the $y$ axis
C. A hyperboloid of one sheet which does not intersect the $z$ axis
D. A hyperboloid of two sheets which does intersect the $y$ axis
E. A hyperboloid of two sheets which does intersect the $z$ axis

The sketch is the best way to see this.

$x^2 + 3y^2 = 4 + 2y$
9. Let \( \mathbf{r}(0) = i + j, \mathbf{v}(0) = 2i + 3j, a(t) = e^{2t}j, \) where \( \mathbf{r}''(t) = a(t) \) and \( \mathbf{r}'(t) = \mathbf{v}(t). \) Find \( \mathbf{r}(1). \)

\[
\mathbf{v}(t) = \int_{0}^{t} a(\omega) \, d\omega + \mathbf{v}(0) \\
= \left[ \int_{0}^{t} e^{2\omega} \, d\omega \right] \mathbf{i} + \left[ \int_{0}^{t} \frac{1}{4} e^{2\omega} \, d\omega \right] \mathbf{j} + 2\mathbf{i} + 3\mathbf{j} \\
= 2\mathbf{i} + \left[ \frac{1}{2} e^{2\omega} \bigg|_{0}^{t} \right] + 3 \mathbf{j} \\
= 2\mathbf{i} + \left[ \frac{1}{2} e^{2t} + \frac{3}{2} \right] \mathbf{j} \\
= 2\mathbf{i} + \left[ \frac{1}{2} e^{2t} + 2\frac{1}{2} \right] \mathbf{j}
\]

\[
\mathbf{r}(t) = \mathbf{r}(0) + \int_{0}^{t} \mathbf{v}(\omega) \, d\omega = \\
= \mathbf{i} + \mathbf{j} + \left[ \int_{0}^{t} 2 \, d\omega \right] \mathbf{i} + \left[ \int_{0}^{t} \left( \frac{1}{2} e^{2\omega} + 2\frac{1}{2} \right) \, d\omega \right] \mathbf{j} \\
= \mathbf{i} \left[ 1 + 2 \right] + \mathbf{j} \left[ 1 + \left. \frac{1}{4} e^{2\omega} \right|_{0}^{t} + 2\frac{1}{2} \left. \right|_{0}^{t} \right] \\
= \mathbf{i} \left[ 1 + \frac{3}{2} \right] + \mathbf{j} \left[ 1 + 2\frac{1}{4} + 2\frac{1}{2} \right] \\
\]

Plug in \( t = 1 \) and select D.
10. If $E$ is the region defined by $y > 0, y - x < 0$, and $x^2 + y^2 + z^2 < 4$, then describe $E$ in spherical coordinates

- A. $0 < \rho < 4, 0 < \theta < \frac{\pi}{2}, 0 < \phi < \pi$
- B. $0 < \rho < 2, 0 < \theta < \frac{\pi}{4}, 0 < \phi < \frac{\pi}{2}$
- C. $0 < \rho < 2, \frac{\pi}{4} < \theta < \pi, 0 < \phi < \frac{\pi}{2}$
- D. $0 < \rho < 2, 0 < \theta < \frac{\pi}{2}, 0 < \phi < \frac{\pi}{4}$
- E. $0 < \rho < 2, 0 < \theta < \frac{\pi}{4}, 0 < \phi < \pi$

The set $\{y > 0, y - x < 0\}$ in the $x y$ plane looks like this:

Since $y - x < 0$ is the part of the plane on the $+x$ side of the line $y - x = 0$.

In polar coordinates this is $0 < \theta < \frac{\pi}{4}$.

In 3D, $\{y > 0, y - x < 0\}$ is everything which projects to the region sketched above, a wedge. $x^2 + y^2 + z^2 < 4 = \rho < 2$, the sphere of radius 2 about the origin.

So the region is a wedge out of a sphere with the sharp edge of the wedge along the $y$-axis from -2 to 2. So every $\phi$ from 0 to $\pi$ is the $\phi$ of a point in the region: (E)
11. The tangent line to the curve traced out by \( \mathbf{r}(t) = (\cos t, \sin t, t) \) at the point \((0, 1, \frac{\pi}{2})\) hits the \(xy\) plane at the point where

\[ t \text{ must be } \frac{\pi}{2} \]

A. \( x = 1, \ y = \pi \)
B. \( x = \frac{\pi}{2}, \ y = 1 \)
C. \( x = \pi, \ y = \frac{\pi}{2} \)
D. \( x = -\frac{\pi}{2}, \ y = 1 \)
E. \( x = -1, \ y = \pi/2 \)

\[ \mathbf{r}(\frac{\pi}{2}) = (0, 1, \frac{\pi}{2}) \]

To find the line use \( \mathbf{r}(\frac{\pi}{2}) \) as the point and \( \mathbf{r}'(\frac{\pi}{2}) = (-\sin t, \cos t, 1) \) evaluated at \( t = \frac{\pi}{2} \) as a parallel vector.

\((0, 1, \frac{\pi}{2}) + t(-1, 0, 1)\) gives tangent line.

This hits the \(xy\) plane when the \(z\) coordinate is 0, so \( \frac{\pi}{2} + t = 0 \), \( t = -\frac{\pi}{2} \)

The first two coordinates of the answer is \( (0, 1, \frac{\pi}{2}) + (-\frac{\pi}{2})(-1, 0, 1) \).

\[ x = 0 + (-\frac{\pi}{2}) \cdot (-1) \]
\[ y = 1 + (-\frac{\pi}{2}) \cdot 0 \]