MA261 — EXAM I — FALL 2014 — OCTOBER 9, 2014
EXAM TYPE I

INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. There are 7 different test pages (including this cover page). Once you are allowed to open the exam, make sure you have a complete test.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet.
4. The number of points each problem is worth is stated next to it. The maximum possible score is 100 points. No partial credit.
5. Make sure the color of your scantron matches the color of the cover page of your exam.
6. Use a # 2 pencil to fill in the required information in your scantron and fill in the circles.
7. Use a # 2 pencil to fill in the answers on your scantron.
8. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY:

1. Do not leave the exam room during the first 20 minutes of the exam.
2. If you do not finish your exam in the first 50 minutes, you must wait until the end of the exam period to leave the room.
3. Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.
4. Do not look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
5. Do not consult notes, books, calculators.
6. Do not handle phones or cameras, or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: ANSWERS.

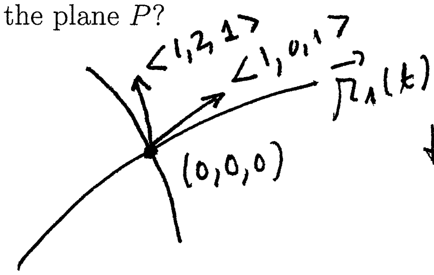
STUDENT SIGNATURE: ______________________

STUDENT ID NUMBER: ______________________

SECTION NUMBER ______________________

RECITATION INSTRUCTOR: ______________________

1. **(8 points)** Two curves $r_1(t)$ and $r_2(t)$ intersect at $(0,0,0)$ when $t = 1$. We know that the tangent vector to $r_1(t)$ at $(0,0,0)$ is $r_1'(1) = \langle 1,0,1\rangle$ and the tangent vector to $r_2(t)$ at $(0,0,0)$ is $r_2'(1) = \langle 1,2,1\rangle$. If a plane P passes through $(0,0,0)$ and contains $r_1'(1)$ and $r_2'(1)$, which of the following is an equation of the plane P?



A. $z + y - 3x = 0$

B. $2z + 4x - 3y = 0$

C. $3x = 2y$

$\boxed{\text{D. } x = z}$

E. $4x + 2z - 7y = 0$

Plane is perpendicular to ~~vectors~~ $\langle 1,0,1\rangle$ and $\langle 1,2,1\rangle$.

Normal to the plane: $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ 1 & 2 & 1 \end{vmatrix} = -2\vec{i} + 2\vec{k}$. $= -2\vec{i}$ $2\vec{k}$.

Equation of the plane:
$\langle -2, 0, 2\rangle \cdot \langle x, y, z\rangle = -2x + 2z = 0 \quad \therefore \boxed{x = z}$

2. **(8 points)** The surface S_1 is represented by $z = x^2 - y^2$, the surface S_2 is represented by $z^2 = x^2 + y^2$, and the surface S_3 is represented by $y = x^2 + 3z$. Which of the following are true?

I. The intersections of S_1 with the plane $x = 1$ is a parabola T
II. The intersection of S_2 with the plane $x = 1$ is a hyperbola T
III. The intersection of S_3 with the plane $x = 1$ is a parabola F

$\boxed{\text{A. I and II are true, but III is false}}$

B. I is true, but II and III are false

C. II and III are true, but I is false

D. III is true, but I and II are false

E. I, II and III are false

S_1: $z = x^2 - y^2$, if $x = 1$
$z = 1 - y^2$. parabola.

S_2: $z^2 = x^2 + y^2$ if $x = 1$
$z^2 - y^2 = 1$ hyperbola.

S_3: $y = x^2 + 3z$ if $x = 1$
$y = 1 + 3z$ line.

3. **(8 points)** Find the length of the curve $r(t) =< 2t, \frac{t^3}{3}, t^2 >$ where t varies from 0 to 1.

A. $\frac{5}{3}$

B. $\boxed{\frac{7}{3}}$

C. $\frac{1}{3}$

D. $\frac{8}{3}$

E. $\frac{4}{3}$

$$L = \int_0^1 |\vec{r}\,'(t)|\,dt \qquad \vec{r}\,'(t) = < 2, t^2, 2t >$$

$$|\vec{r}\,'(t)| = \sqrt{4 + t^4 + 4t^2} = \sqrt{(t^2+2)^2} = t^2 + 2.$$

$$L = \int_0^1 (t^2+2)\,dt = 2 + 1/3 = 7/3$$

4. **(8 points)** Compute $\left| \int_0^1 < t^{\frac{3}{2}}, t, 1 > dt \right|$ (the length of the vector).

A. $\frac{\sqrt{136}}{10}$

B. $\frac{\sqrt{139}}{10}$

C. $\boxed{\frac{\sqrt{141}}{10}}$

D. $\frac{\sqrt{113}}{10}$

E. $\frac{\sqrt{123}}{10}$

$$\int_0^1 < t^{3/2}, t, 1 > dt = < \tfrac{2}{5} t^{5/2}, \tfrac{t^2}{2}, t > \Big|_0^1$$

$$= < 2/5, 1/2, 1 >.$$

$$|< 2/5, 1/2, 1 >|^2 = 1 + \frac{1}{4} + \frac{4}{25} = \frac{100+25+16}{100}$$

$$= \frac{141}{100}.$$

$$\left| \int_0^1 < t^{3/2}, t, 1 > dt \right| = \frac{\sqrt{141}}{10}$$

5. **(8 points)** The level curves of $z = \sqrt{9 - x^2 - y^2}$ are:

A. Concentric circles and points

B. Parabolas

C. Ellipses

D. Lines and points

E. Circles (which do not necessarily have the same center) and points

$z = c$:

$9 - x^2 - y^2 = c^2$

$x^2 + y^2 = 9 - c^2$

If $c^2 < 9$ Circles centred at $(0,0)$

$c^2 = 9$. Point.

$c > 0$. No level curve

6. **(8 points)** The function $f(x, y) = x^3 - 6xy + y^3$ has critical points $(0, 0)$ and $(2, 2)$. At these points f has:

A. two relative minima

B. a saddle point and a relative minimum

C. a saddle point and a relative maximum

D. two saddle points

E. two relative maxima

$\frac{\partial f}{\partial x} = 3x^2 - 6y$

$\frac{\partial^2 f}{\partial x^2} = 6x$, $\frac{\partial^2 f}{\partial x \partial y} = -6$

$\frac{\partial f}{\partial y} = 3y^2 - 6x$

$\frac{\partial^2 f}{\partial y^2} = 6y$

$D = 36xy - 36$ $\quad D(0,0) = -36$ Saddle

$D(2,2) = 4 \times 36 - 36 = 3 \times 36 = 108 > 0.$

$f_{xx}(2,2) = 12 > 0$ Min.

7. **(8 points)** The volume of a cylinder satisfies $V = \pi r^2 h$. At a certain instant the cylinder has radius $r = 2$ in, height $h = 3$ in., the radius increases at the rate of 1 in/min., and the volume increases at the rate of 4π in.^{3}/min. Determine the rate of change of h.

A. increases at the rate of 2 in./min.

$\boxed{\text{B. decreases at the rate of 2 in./min.}}$

C. increases at the rate of π in./min.

D. decreases at the rate of 2π in./min.

E. decreases at the rate of 3 in./min.

$V = \pi r^2 h$ $\qquad \frac{dr}{dt} = 1$ in/min

$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}.$ $\qquad \frac{dV}{dt} = 4\pi$ in^3/min

when $r = 2$, $h = 3$

$4\pi = 2\pi \cdot 2 \cdot 1 \cdot 3 + 4\pi \frac{dh}{dt}$ $\therefore$

$4\pi = 12\pi + 4\pi \frac{dh}{dt}$; $\boxed{\frac{dh}{dt} = -2 \text{ in/min}}$

8. **(8 points)** For which direction $\mathbf{u}$ will the directional derivative of $f(x, y) = xy^{-2}$ at the point $(2, 1)$ have the value 0?

A. $\mathbf{u} = \langle 1, -4 \rangle$

B. $\mathbf{u} = \langle 1, 4 \rangle$

C. $\mathbf{u} = \langle \frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \rangle$

D. $\mathbf{u} = \langle \frac{-1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \rangle$

$\boxed{\text{E. } \mathbf{u} = \langle \frac{4}{\sqrt{17}}, \frac{1}{\sqrt{17}} \rangle}$

$\vec{u} = u_1 \vec{i} + u_2 \vec{j}$; $|\vec{u}| = 1$

$D_u f = u_1 \frac{\partial f}{\partial x} + u_2 \frac{\partial f}{\partial y}$ $\qquad \frac{\partial f}{\partial x} = y^{-2}$

$\frac{\partial f}{\partial y} = -2xy^{-3}$

$\frac{\partial f}{\partial x}(2,1) = 1$

$\frac{\partial f}{\partial y}(2,1) = -4.$

$D_u f = u_1 - 4u_2 = 0.$ $\qquad u_1 = 4u_2.$

$u_1^2 + u_2^2 = 1.$

9. **(8 points)** If $u(x,y) = \ln(x^2y^4) + 3x^2e^{2y}$, in $\{x > 0, \ \ y > 0\}$, find $\dfrac{\partial u}{\partial y} + \dfrac{\partial^2 u}{\partial y \partial x}$ at $(1,1)$

A. $\boxed{4 + 18e^2}$

B. $-4 + 9e^2$

C. $1 + 10e^2$

D. $4 - 6e^2$

E. $2 - 10e^2$

$$u = 2\ln x + 4\ln y + 3x^2e^{2y}$$

$$\frac{\partial u}{\partial x} = 2/x + 6xe^{2y}.$$

$$\frac{\partial^2 u}{\partial y \partial x} = 12xe^{2y} \ ; \ \frac{\partial^2 u}{\partial y\partial x}(1,1) = 12e^2.$$

$$\frac{\partial u}{\partial y} = 4/y + 6x^2e^{2y} \quad \frac{\partial u}{\partial y}(1,1) = 4 + 6e^2$$

$$\boxed{\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y\partial x} = 4 + 18e^2}$$

10. **(8 points)** If $z(x,y)$ is defined implicitly by the equation

$$6e^z + yz^2 + xy^2 - x^3 = 0,$$

find $\dfrac{\partial z}{\partial x}(x,y)$ at $(x_0, y_0, z_0) = (2, -1, 0)$.

A. $\dfrac{5}{3}$

B. $\boxed{\dfrac{11}{6}}$

C. 2

D. $\dfrac{1}{2}$

E. $-\dfrac{5}{3}$

$$6e^z\frac{\partial z}{\partial x} + 2yz\frac{\partial z}{\partial x} + y^2 - 3x^2 = 0.$$

When $x = 2, y = -1, \ z = 0$

$$6\frac{\partial z}{\partial x} + 1 - 12 = 0$$

$$\boxed{\partial z/\partial x = 11/6}$$

11. **(10 points)** If $f(x, y) = x\ln(x^2 + y^2)$, and $x = t^2 + 1$, $y = 2t + 1$, find $\frac{df}{dt}$ at $(x, y) = (2, -1)$.

A. $2\ln 5 - \frac{8}{5}$

B. $-2\ln 5 - \frac{24}{5}$

C. $2\ln 5 - \frac{16}{5}$

D. $2\ln 5 - \frac{4}{5}$

E. $2\ln 5 + \frac{12}{5}$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$\frac{\partial f}{\partial t} = \left[\ln(x^2+y^2) + \frac{2x^2}{x^2+y^2}\right]\cdot 2t + \frac{2xy}{x^2+y^2}\cdot 2.$$

$x=2,\ y=-1$

$$\frac{\partial f}{\partial t} = \left[\ln 5 + \frac{8}{5}\right](-2) - \frac{8}{5}$$

$$= -2\ln 5 - \frac{24}{5}$$

$2t+1=-1$

$2t=-2$

$t=-1$

EVERYONE RECEIVED 10PTS FOR THIS QUESTION

12. **(10 points)** Which of the points $(0, 0, 0)$, $(1, 1, 2)$ and $(0, 1, 2)$ lie on the tangent plane to the surface $z = x^4 + 2x^2y + y^2 - 3$ at $(-1, 0, 4)$?

A. Only $(0, 0, 0)$

B. Only $(0, 0, 0)$ and $(1, 1, 2)$

C. Only $(1, 1, 2)$ and $(0, 1, 2)$

D. Only $(1, 1, 2)$

E. Only $(0, 0, 0)$ and $(0, 1, 2)$

There was a typo in this question. It should have said:

$$z = x^4 + 2x^2y + y^2 + 3.$$

In this case

$$z - x^4 - 2x^2y - y^2 = 3$$

Has normal vector at $(-1, 0, 4)$

$$\langle -4x^3 - 4xy, -2x^2 - 2y, 1\rangle\Big|_{\substack{x=-1\\ y=0\\ z=4}} = \langle 4, -2, 1\rangle$$

Plane is: $\langle 4,-2,1\rangle\cdot\langle x+1, y, z-4\rangle = 0$ $\quad 4x+4-2y+z-4=0$

$$\boxed{z+4x-2y=0}$$

$(0,0,0)$ and $(0,1,2)$ lie on this plane $(1,1,2)$ does not.