NAME Solution

STUDENT ID # ________________

RECITATION INSTRUCTOR ________________

RECITATION TIME ________________

DIRECTIONS

1) Fill in the above information. Also write your name at the top of each page of the exam.
2) The exam has 6 pages, including this one.
3) Problems 1 through 6 are multiple choice; circle the correct answer. No partial credit for these problems.
4) Problems 7 through 9 are problems to be worked out. Partial credit for correct work is possible. Write your answer in the box provided. YOU MUST SHOW SUFFICIENT WORK TO JUSTIFY YOUR ANSWERS. CORRECT ANSWERS WITH INCONSISTENT WORK MAY NOT RECEIVE CREDIT.
5) Points for each problem are given in parenthesis in the left margin.
6) No books, notes, or calculators may be used on this test.

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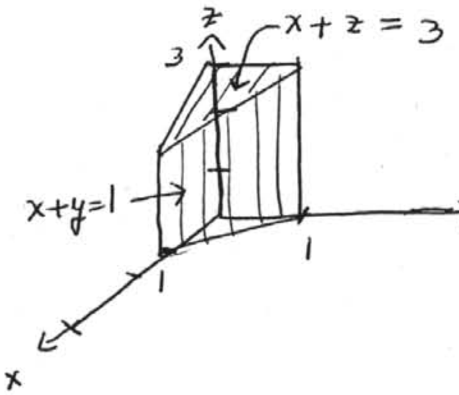
TOTAL	/100

(8) 1. Find the volume of the solid region in the first octant bounded above by the plane $x + z = 3$, on the sides by the planes $x + y = 1$, $x = 0$, and $y = 0$ and below by the plane $z = 0$..



$$V = \int_0^1 \int_0^{1-x} \int_0^{3-x} dz\, dy\, dx$$

$$= \int_0^1 (1-x)(3-x)\, dx$$

$$= \int_0^1 (3 - 4x + x^2)\, dx = 3x - 2x^2 + \tfrac{1}{3}x^3 \Big]_0^1$$

$$= \frac{4}{3}$$

A. $\frac{5}{3}$

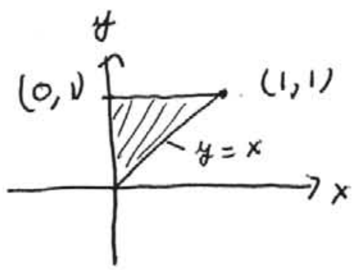
B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{4}{3}$

E. 2

(8) 2. Find the surface area of the part of the parabolic cylinder $z = y^2$ that lies over the triangle with vertices $(0,0)$, $(0,1)$, $(1,1)$ in the xy-plane.



$0 \le x \le y$

$0 \le y \le 1$

$$F(x,y,z) = y^2 - z$$

$$\frac{\partial F}{\partial x} = 0, \quad \frac{\partial F}{\partial y} = 2y$$

$$\frac{\partial F}{\partial z} = -1$$

$$ds = \sqrt{\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2 + \left(\frac{\partial F}{\partial z}\right)^2}\, dx\,dy$$

$$= \sqrt{4y^2 + 1}\, dx\,dy$$

A. $\frac{1}{12}(5\sqrt{5} - 1)$

B. $\frac{5}{12}\sqrt{5}$

C. $\frac{2}{3}$

D. $\frac{1}{4}$

E. $\frac{1}{12}$

$$S = \int_0^1 \int_0^y \sqrt{4y^2+1}\, dx\, dy$$

$$= \int_0^1 y\sqrt{4y^2+1}\, dy = \frac{1}{8}\int_0^1 8y\,(4y^2+1)^{1/2}\, dy$$

$$= \frac{1}{8} \cdot \frac{2}{3} (4y^2+1)^{3/2} \Big]_0^1 = \frac{1}{12}\left(5^{3/2} - 1\right)$$

(8) 3. The iterated triple integral

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{x}^{3+x^2+y^2} 10y\,dz\,dy\,dx$$

in cylindrical coordinates is:

A. $\int_0^{\pi}\int_0^2\int_{\cos\theta}^{3+r^2} 10r\sin\theta dz\,dr\,d\theta$

B. $\int_0^{\frac{\pi}{2}}\int_0^2\int_{r\cos\theta}^{3+r^2} 10r\sin\theta dz\,dr\,d\theta$

C. $\int_0^{\pi}\int_{-2}^2\int_{r\cos\theta}^{3+r^2} 10r^2\sin\theta dz\,dr\,d\theta$

D. $\int_0^{\pi}\int_0^2\int_{r\cos\theta}^{3+r^2} 10r^2\sin\theta dz\,dr\,d\theta$

E. $\int_0^{\frac{\pi}{2}}\int_{-2}^2\int_{r\cos\theta}^{3+r^2} 10r^2\sin\theta dz\,dr\,d\theta$

$y = r\sin\theta$

$dA = r\,dr\,d\theta$

$x = r\cos\theta$

$3 + x^2 + y^2 = 3 + r^2$

$y = \sqrt{4-x^2}$ is the semicircle $r = 2,\ 0 \le \theta \le \pi$

$-2 \le \quad \le 2$

$0 \le \theta \le \pi$

(8) 4. If $\vec{F}(x,y,z) = xy\vec{i} + z^2\vec{j} + e^y\vec{k}$ then $\vec{F}\cdot \text{curl}\,\vec{F} =$

A. 0

B. xy^2

C. $xy(e^y - 2z) - xe^y$

D. $e^y(xy+2z-x) - yz^2$

E. $e^y(xy+2z-x)$

$$\text{curl}\,\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & z^2 & e^y \end{vmatrix}$$

$$= \left(\frac{\partial e^y}{\partial y} - \frac{\partial z^2}{\partial z}\right)\vec{i} + \left(\frac{\partial xy}{\partial z} - \frac{\partial e^y}{\partial x}\right)\vec{j} + \left(\frac{\partial z^2}{\partial x} - \frac{\partial xy}{\partial y}\right)\vec{k}$$

$$= (e^y - 2z)\vec{i} + 0\vec{j} - x\vec{k} = (e^y - 2z)\vec{i} - x\vec{k}$$

$$\vec{F}\cdot \text{curl}\,\vec{F} = xy(e^y - 2z) + z^2\cdot 0 - xe^y$$

$$= xy(e^y - 2z) - xe^y$$

(8) 5. Compute $\int_C 6x\,ds$ where C is the graph of $y = x^2$ for $0 \le x \le 1$.

A. $5\sqrt{5} - 1$

B. $\frac{1}{2}(5\sqrt{5} - 1)$

C. 3

D. 2

E. $\frac{3}{2}$

$y = x^2$ can be paramatized as

$x(t) = t, \quad y(t) = t^2, \quad z(t) = 0, \; 0 \le t \le 1$

$$ds = \frac{dr}{dt}dt = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt$$

$$= \sqrt{1 + 4t^2}\,dt$$

$$\therefore \int_C 6x\,dx = \int_0^1 6t\sqrt{1+4t^2}\,dt$$

$$= \frac{3}{4}\int_0^1 8t(1+4t^2)^{1/2}\,dt$$

$$= \frac{3}{4}\cdot\frac{2}{3}(1+4t^2)^{3/2}\Big]_0^1 = \frac{1}{2}(5^{3/2} - 1)$$

(8) 6. Compute $\int_C e^x\,dx + 3xy\,dy + xyz\,dz$ where C is the curve parametrized by $\vec{r}(t) = t\vec{i} + t\vec{j} + 2t\vec{k}$ for $0 \le t \le 1$.

A. e

B. $e + \frac{1}{3}$

C. $e + \frac{1}{2}$

D. $e + 1$

E. $e + \frac{3}{2}$

$$I = \int_0^1 (e^t \cdot 1 + 3t^2 \cdot 1 + 2t^3 \cdot 2)\,dt$$

$$= \int_0^1 (e^t + 3t^2 + 4t^3)\,dt$$

$$= e^t + t^3 + t^4\Big]_0^1$$

$$= (e + 1 + 1) - e^0$$

$$= e + 1$$

(11) 7. Find a function $f(x,y)$ whose gradient is:
grad $f(x,y) = (3x^2e^{2y} - y)\vec{i} + (2x^3e^{2y} - x + 2y)\vec{j}$ and $f(1,0) = 3$.

$$\frac{\partial f}{\partial x} = 3x^2e^{2y} - y$$

$$f(x,y) = x^3e^{2y} - xy + h(y)$$

$$\frac{\partial f}{\partial y} = 2x^3e^{2y} - x + 2y, \quad \frac{\partial f}{\partial y} = 2x^3e^{2y} - x + h'(y)$$

$$h'(y) = 2y, \quad h(y) = y^2 + C$$

$$3 = f(1,0) = 1 + C \quad \therefore \quad C = 2$$

$f(x,y) =$ $\boxed{x^3e^{2y} - xy + y^2 + 2}$

(11) 8. Use Green's Theorem to evaluate $\int_C (y^3 + 2y)dx + 3xy^2dy$, where C is the circle $x^2 + y^2 = 16$ oriented counterclockwise.

$$\frac{\partial N}{\partial x} = 3y^2, \quad \frac{\partial M}{\partial y} = 3y^2 + 2$$

$$\int_C (y^3 + 2y)dx + 3xy^2dy = \iint_R -2\,dA$$

$$= \int_0^{2\pi}\int_0^4 -2r\,dr\,d\theta$$

or $= -2(\text{area of circle})$

$\boxed{-32\pi}$

(30) 9. Let D be the solid region above the upper nappe of the cone $z^2 = x^2 + y^2$ and below the sphere $x^2 + y^2 + z^2 = 18$. If $\vec{F}(x, y, z) = 3x^2\vec{i} + \vec{j} + \frac{1}{2}z^2\vec{k}$, express the triple $\iiint_D \operatorname{div} \vec{F}\, dV$ as an iterated triple integral in (a) rectangular coordinates, (b) cylindrical coordinates, and (c) spherical coordinates. (Include the limits of integration.)

(a) Rectangular coordinates

$\operatorname{div} \vec{F} = 6x + z$

or $\displaystyle\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} (6x+z)\,dz\,dx\,dy.$

$$\boxed{\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} (6x+z)\,dz\,dy\,dx}$$

(b) Cylindrical coordinates

$\operatorname{div} \vec{F} = 6r\cos\theta + z$

$$\boxed{\int_{0}^{2\pi}\int_{0}^{3}\int_{r}^{\sqrt{18-r^2}} (6r\cos\theta + z)\,r\,dz\,dr\,d\theta}$$

(c) Spherical coordinates

$\operatorname{div} \vec{F} = 6\rho\sin\phi\cos\theta + \rho\cos\phi$

$$\boxed{\int_{0}^{2\pi}\int_{0}^{\pi/4}\int_{0}^{\sqrt{18}} (\operatorname{div} \vec{F})\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta}$$