SOLUTION

MATH 261 - SPRING 2001

FINAL EXAM

Name ______________________ Instructor ______________________

Signature ______________________ Recitation Instructor ______________________

Div. Sect. No. ______________________

FINAL EXAM INSTRUCTIONS

1. You must use a #2 pencil on the mark-sense sheet (answer sheet).
2. On the mark-sense sheet, fill in the instructor's name and the course number.
3. Fill in your name and student identification number and blacken in the appropriate spaces.
4. Mark in your division and section number of your class. For example, for division 02, section 03, fill in 0203 and blacken the corresponding circles, including the circles for the zeros. (If you do not know your division and section number ask your instructor.)
5. Sign the mark-sense sheet.
6. Fill in the information above and fill in your name on each of the question sheets.
7. There are 20 questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20. Do all your work on the question sheets. Turn in both the mark-sense sheets and the question sheets when you are finished.
8. No partial credit will be given, but if you show your work on the question sheets it may be considered if your grade is on the borderline.
9. Calculators are not allowed. NO BOOKS OR PAPERS ARE ALLOWED. Use the back of the test pages for scratch paper.

Name ______________________________

1. Give the equation of a plane containing the points $(1, 0, 0)$ and $(1, 2, 1)$ and parallel to the line whose equations are $x = y$, $z = 0$.

A. $x + y = 1$

B. $x - y = 1$

C. $x - y + z = 1$

D. $x - y + 2z = 1$

E. $x + y - z = 1$

2. Let $\vec{r}(t) = e^t \cos t \vec{\imath} + e^t \sin t \vec{\jmath}$ be parametric equations of a curve C. Find the length of C from $t = 0$ to $t = \pi$.

A. $\sqrt{2}\, e^{\pi}$

B. $\sqrt{2}\, (e^{\pi} - e^{-\pi})$

C. $\sqrt{2}\, (e^{\pi} - 1)$

D. $\sqrt{2}\, (e^{\pi} + e^{-\pi})$

E. $\sqrt{2}\, (e^{\pi} + 1)$

Name ______________________________

3. If $f(x,y) = 5 - 4x^3 + 8y^2$, find a unit vector $\vec{u}$ for which the directional derivative $D_{\vec{u}}f(1,1)$ is zero.

A. $\frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}$

B. $\frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}$

C. $-\frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}$

D. $-\frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}$

E. $\frac{4}{5}\vec{i} - \frac{3}{5}\vec{j}$

4. Compute $\frac{\partial z}{\partial r} + \frac{\partial z}{\partial s}$ if $z = \sin u \cos v$, $u = (r+s)^2$, and $v = (r-s)^2$.

A. $4(r+s)\sin(r+s)^2\cos(r-s)^2$

B. $4(r-s)\cos(r+s)^2\sin(r-s)^2$

C. $4(r+s)\cos(r+s)^2\cos(r-s)^2$

D. $4(r-s)\cos(r+s)^2\cos(r-s)^2$

E. $4(r+s)\sin(r+s)^2\sin(r-s)^2$

Name ______________________________

5. The maximum value of $f(x, y) = 2x + 4y$ on the circle $x^2 + y^2 = 5$ is:

A. /6

B. 8

C. 10

D. $4\sqrt{5}$

E. $6\sqrt{10}$

6. The function $f(x, y) = x^3 + y^3 + 3xy$ has critical points $(0, 0)$ and $(-1, -1)$. These critical points are

A. both maximum points

B. both minimum points

C one minimum and one maximum point

D. one minimum and one saddle point

E. one maximum and one saddle point

Name ______________________________

7. If the order of integration is reversed, which of the following integrals is equal to $\int_0^{\sqrt{\pi}} \int_{y^2}^{\pi} (\sin x^2) dx\, dy$?

A. $\int_0^{\pi} \int_{\sqrt{x}}^{\pi} (\sin x^2) dy\, dx$

B. $\int_0^{\pi} \int_0^{\sqrt{x}} (\sin x^2) dy\, dx$

C. $\int_{\sqrt{x}}^{\sqrt{\pi}} \int_0^{\pi} (\sin x^2) dy\, dx$

D. $\int_0^{\pi} \int_x^{\sqrt{\pi}} (\sin x^2) dy\, dx$

E. $\int_0^{\sqrt{\pi}} \int_x^{\pi} (\sin x^2) dy\, dx$

8. If R is the region in the xy-plane inside the circle $x^2 + y^2 = 1$ and above the line $y = x$, then $\iint_R x\, dA$ expressed in polar coordinates is:

A. $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^1 r \cos\theta dr\, d\theta$

B. $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^1 r^2 \cos\theta dr\, d\theta$

C. $\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \int_0^{r\cos\theta} r^2 \cos\theta dr\, d\theta$

D. $\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \int_0^{r^2} r^2 \cos\theta dr\, d\theta$

E. $\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \int_0^1 r^2 \cos\theta dr\, d\theta$

Name ______________________________

9. The surface area of the portion of the parabolic sheet $z = 1 - x^2$ bounded on the sides by the planes $y = 0$, $x = 2$, and $y = 4x$ is:

A. 1

B. $\frac{1}{3}(5^{\frac{3}{2}} - 1)$

C. $5^{\frac{3}{2}} - 1$

(D.) $\frac{1}{3}(17^{\frac{3}{2}} - 1)$

E. $17^{\frac{3}{2}} - 1$

10. An object occupies the tetrahedron in the first octant bounded by the coordinate planes and the plane $x + 2y + z = 2$. Express the total mass of the object as a triple integral if the mass density at each point is the distance from the point to the yz-plane.

A. $\int_0^1 \int_0^{-2y} \int_0^{2-x-2y} x\,dz\,dx\,dy$

B. $\int_0^2 \int_0^{2-2y} \int_0^{2-x-2y} x\,dz\,dx\,dy$

(C.) $\int_0^1 \int_0^{2-2y} \int_0^{2-x-2y} x\,dz\,dx\,dy$

D. $\int_0^1 \int_0^{-2y} \int_0^{2-x-2y} (2-2y-z)dz\,dx\,dy$

E. $\int_0^1 \int_0^{2-2y} \int_0^{2-x-2y} (2-2y-z)dz\,dx\,dy$

Name ______________________________

11. If D is the solid region in the first octant bounded above by the paraboloid $z = 1 - x^2 - y^2$ and below by the xy-plane, the volume of D is:

A. $\frac{\pi}{8}$

B. $\frac{\pi}{12}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{3}$

E. $\frac{\pi}{4}$

12. The integral $\int_0^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{\sqrt{3x^2+3y^2}}^{\sqrt{8-x^2-y^2}} xyz\, dz\, dy\, dx$ in cylindrical coordinates is:

A. $\int_0^{\frac{\pi}{2}} \int_0^{\sqrt{2}} \int_{\sqrt{3}r}^{\sqrt{8-r^2}} r^3 z \cos\theta \sin\theta dz\, dr\, d\theta$

B. $\int_0^{\frac{\pi}{2}} \int_0^{2} \int_{\sqrt{3}r}^{\sqrt{8-r^2}} r^2 z \cos\theta \sin\theta dz\, dr\ \ d\theta$

C. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\sqrt{2}} \int_{\sqrt{3}r}^{\sqrt{8-r^2}} r^3 z \cos\theta \sin\theta dz\, dr\, d\theta$

D. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2} \int_{\sqrt{3}r}^{\sqrt{8-r^2}} r^3 z \cos\theta \sin\theta dz\, dr\, d\theta$

E. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2} \int_{\sqrt{3}r}^{\sqrt{8-r^2}} r^2 z \cos\theta \sin\theta dz\, dr\, d\theta$

13. The mass of an object occupying the region bounded above by the plane $z = 2$ and below by the upper nappe of the cone $z^2 = x^2 + y^2$ with mass density at each point equal to $x^2 + y^2 + z^2$ is given by:

A. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

(B.) $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{2\sec\phi} \rho^4 \sin\phi \, d\rho \, d\phi \, d\theta$

C. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^2 \rho^4 \sin\phi \, d\rho \, d\phi \, d\theta$

D. $\int_0^{2\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_0^{2\sec\phi} \rho^3 \sin^2\phi \, d\rho \, d\phi \, d\theta$

E. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{2\sec\phi} \rho^3 \sin^2\phi \, d\rho \, d\phi d\theta$

14. Let $\vec{F}(x, y, z) = (z + y^2)\vec{i} + 2xy\vec{j} + (x + y)\vec{k}$. Find the curl($\vec{F}$) at the point $(1, 1, 1)$.

A. $\vec{i} + \vec{j} + \vec{k}$

B. $\vec{j}$

(C.) $\vec{i}$

D. $-\vec{i}$

E. $-\vec{j}$

Name ______________________________

15. The area of the region in the first quadrant which lies outside the circle $r = 1$ and inside the circle $r = 2\cos\theta$ is given by:

A. $\int_0^{\frac{\pi}{3}} \int_1^{2\cos\theta} r\, dr\, d\theta$

B. $\int_0^{\frac{\pi}{3}} \int_0^{2\cos\theta} r\, dr\, d\theta$

C. $\int_0^{\frac{\pi}{2}} \int_1^{2\cos\theta} r^2\, dr\, d\theta$

D. $\int_0^{\frac{\pi}{2}} \int_0^{2\cos\theta} r\, dr\, d\theta$

E. $\int_0^{\pi} \int_0^{2\cos\theta} r^2\, dr\, d\theta$

16. If C is the curve $y = \frac{x^2}{2} + 1$ from $(0,1)$ to $(2,3)$, then $\int_C 3x\, ds =$

A. $\frac{8}{3}$

B. $\frac{10}{3}$

C. $\sqrt{5}$

D. $\sqrt{5} - 1$

E. $5\sqrt{5} - 1$

Name ________________________________

17. If $\vec{F}(x,y) = (2xe^y + 1)\vec{i} + (x^2e^y)\vec{j}$ is a conservative vector field, that is $\vec{F}(x,y) = \text{grad } f(x,y)$ for some function f, and C is any smooth curve from $(0,0)$ to $(1,1)$ then $\int_C \vec{F} \cdot d\vec{r} =$

A. 0

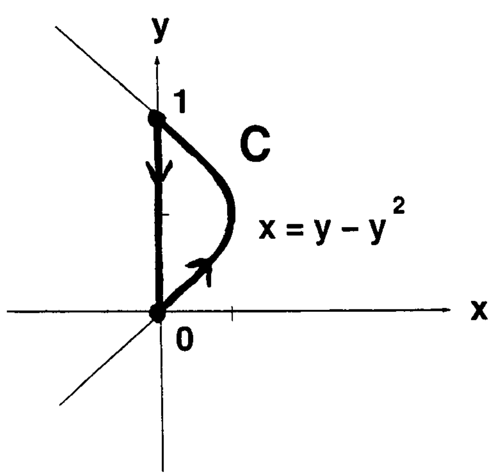
B. 1

C. $2e$

D. $e+1$

E. $2e+1$

18. If C is the oriented closed curve shown below, then $\int_C (e^{2x} + y^2)dx + (14xy + y^2)dy =$



A. 0

B. $\frac{3}{15}$

C. $\frac{7}{30}$

D. 1

E. 2

Name ______________________________

19. Compute the surface integral $\iint_{\Sigma} 4z\,dS$, where Σ is the part of the sphere $x^2+y^2+z^2 = 10$ which lies above the plane $z = 1$.

A. $4\sqrt{10}\,\pi$

(B.) $36\sqrt{10}\,\pi$

C. $40\sqrt{10}\,\pi$

D. 19π

E. 99π

20. Let D be the solid bounded by the cone $z = \sqrt{x^2+y^2}$ and the plane $z = 1$ whose boundary surface Σ is oriented by the unit normal $\vec{n}$ directed outward from D. If $\vec{F}(x,y,z) = (8xz)\vec{i} + (z^3e^{-x})\vec{j} + (y\cos x)\vec{k}$ then $\iint_{\Sigma} \vec{F}\cdot\vec{n}\,dS =$

A. 0

B. π

C. $\frac{\pi}{3}$

(D.) 2π

E. $\frac{16\pi}{3}$