PROBLEM OF THE WEEK
Solution of Problem No. 1 (Fall 2000 Series)

Problem: Given that $\cos 36° = \frac{1}{4} + \frac{1}{4}\sqrt{5}$, show that $(\tan^2 18°)(\tan^2 54°)$ is rational.

Solution (by Mike Hamburg, 11th Grade, St. Joseph H.S., South Bend)

Since $\cos 36° = \frac{1}{4}(\sqrt{5}+1)$, $\cos 72° = 2\cos^2 36° - 1 = \frac{1}{8}(\sqrt{5}+1)^2 - 1 = \frac{1}{4}(\sqrt{5}-1)$. Then

$$(\tan^2 54°)(\tan^2 18°) = \frac{(\sin^2 54°)(\sin^2 18°)}{(\cos^2 54°)(\cos^2 18°)} = \left(\frac{\frac{\cos 36° - \cos 72°}{2}}{\frac{\cos 36° + \cos 72°}{2}}\right)^2$$

$$= \left(\frac{\frac{\sqrt{5}+1}{4} - \frac{\sqrt{5}-1}{4}}{\frac{\sqrt{5}+1}{4} + \frac{\sqrt{5}-1}{4}}\right)^2 = \left(\frac{1}{\sqrt{5}}\right)^2 = \frac{1}{5} \text{ rational.}$$

Also solved by:

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