PROBLEM OF THE WEEK
Solution of Problem No. 10 (Fall 2000 Series)

Problem: Given a rational number $\frac{p}{q}$, show that the equation $\frac{1}{x}+\frac{1}{y}=\frac{p}{q}$ has only finitely many positive integer solutions.

Solution (by Steven Landy, Fac. Physics at IUPUI)

WLOG may assume $\frac{1}{x} \geq \frac{1}{y}$. Then $\frac{1}{x} \geq \frac{1}{2} \cdot \frac{p}{q}$, so $x \leq \frac{2q}{p}$. There are only finitely many positive integers that are no larger than $\frac{2q}{p}$, and since with each solution x, there is only one y, there are only finitely many solutions.

Also solved by:

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