PROBLEM OF THE WEEK
Solution of Problem No. 3 (Fall 2000 Series)

**Problem:** If a given equilateral triangle \( \triangle \) of side \( a \) can be covered by five equilateral triangles of side \( b \), show that \( \triangle \) can be covered by four of side \( b \).

**Solution** (by Mike Hamburg, Jr. St. Joseph H.S., South Bend)

Suppose \( \triangle \) can be covered by 5 equilateral triangles of side \( b \) (henceforth “\( b \)-triangles”). Then we assert \( a \leq 2b \). For if \( a > 2b \), then the vertices and midpoints of the sides of \( \triangle \) (6 points at all) are mutually separated by \( \frac{1}{2}a > b \). But no 2 points on a \( b \)-triangle are separated by a distance greater than \( b \), hence no \( b \)-triangle can cover more than one of the 6 points.

But if \( a \leq 2b \) then 4 \( b \)-triangles can be arranged to form a \( 2b \)-triangle which covers \( \triangle \).

Also solved by:

**Undergraduates:** Kevin Darkes (Soph. A&AE), Haldun Kufluoglu (Sr. EE), James Lee (Sr. MA/CS), Stevie Schraudner (Jr. CS)

**Graduates:** Gajath Gunatillake (MA), Chris Lomont (MA)

**Faculty & Staff:** Steven Landy (Phys. at IUPUI)

**Others:** Damir Dzhafarov, Jake Foster (Sr. & Soph., resp., Harrison H.S., WL)

There was one incorrect solution.