Problem: Prove that the polynomials
\[ P_n(x) = x^{2n} - 2x^{2n-1} + 3x^{2n-2} - \cdots - 2nx + 2n + 1 \quad (n = 1, 2, \cdots) \] have no real zero.

Solution (by Vikram Buddhi, Gr. Math)

Clearly \( P_n(x) > 0 \) for \( x \leq 0 \), since the terms with negative coefficients are multiplied by odd powers of \( x \). Now

\[ P_n(x) + xP_n(x) = x(x^{2n} - x^{2n-1} + x^{2n-2} \cdots - x + 1) + 2n + 1 \]

so

\[ P_n(x) > x^{2n+1} + 1 \quad x + 1 \]
\[ P_n(x) > 2n + 1 \quad \text{for} \quad x > 0. \]

Also solved by:

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Two unacceptable solutions were received.