Problem No. 10 (Fall 2003 Series)

The Lucas numbers are defined as $L_0 = 2$, $L_1 = 1$, $L_{n+2} = L_{n+1} + L_n$ for $n \geq 0$. Find a closed form for the sum

$$\sum_{k=0}^{n} L_k^2$$

in terms of the $L_n$.