PROBLEM OF THE WEEK

Solution of Problem No. 13 (Fall 2003 Series)

Problem: Determine the supremum and infimum of $C(\alpha, \beta, \gamma) = \cos 2\alpha + \cos 2\beta + \cos 2\gamma$, where α, β, γ are the angles of a triangle.

Solution (by Dr. Troy Siemers, Fac. Virginia Military Inst., Lexington, VA)

Supremum is 3, infimum is -1.5.

Since cosine is bounded above by one, C is bounded by 3. But, the (degenerate) triangle with $\alpha = \beta = 0$, $\gamma = \pi$ gives $C(0, 0, \pi) = 3$, so this is the supremum.

Since α, β, γ are the angles of a triangle, $\gamma = \pi - \alpha - \beta$, we can write C as

$$C(\alpha, \beta, \pi - \alpha - \beta) = \cos(2\alpha) + \cos(2\beta) + \cos(2(\pi - \alpha - \beta)).$$

Setting the α and β partial derivatives of C equal to 0, we see that the only other critical point occurs at $(\pi/3, \pi/3, \pi/3)$ to give an infimum of $C(\pi/3, \pi/3, \pi/3) = -1.5$.

Also solved by:

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