PROBLEM OF THE WEEK

Solution of Problem No. 10 (Fall 2006 Series)

Problem:

Prove that for every positive integer n, we have

$$\sum_{k=1}^{n} \frac{1}{k}\left(\binom{n}{k} + 1 \right) = \sum_{k=1}^{n} \frac{2^k}{k}.$$

Solution (by Georges Ghosn and B. Jeevanesan, edited the Panel)

The sum of the first n terms of the geometric series $\sum (1+x)^k$ gives:

$$1 + (x+1) + \cdots + (x+1)^{n-1} = \frac{(x+1)^n - 1}{x+1-1} = \frac{(x+1)^n - 1}{x} = \sum_{k=1}^{n} \binom{n}{k} x^{k-1}. \qquad (1)$$

Integrating both sides of (1) from 0 to 1 yields:

$$\sum_{k=1}^{n} \frac{2^k - 1}{k} = \sum_{k=1}^{n} \binom{n}{k} \frac{1}{k} \Rightarrow \sum_{k=1}^{n} \frac{2^k}{k} = \sum_{k=1}^{n} \frac{1}{k}\left(\binom{n}{k} + 1 \right).$$

At least partially solved by:

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