PROBLEM OF THE WEEK

Solution of Problem No. 12 (Fall 2008 Series)

Problem: Let $1 < n_1 \leq n_2 \leq \ldots$ be a sequence of positive integers such that (i) no n_j is prime and (ii) $(n_i, n_j) = 1$ if $i \neq j$ (i.e., n_i and n_j are relatively prime). Show that $\sum_{j=1}^{\infty} \frac{1}{n_j} < \infty$.

Solution (by Prithwijit De, Kolkata, India)

For $j \geq 1$ let p_j be the smallest prime divisor of n_j. Since $(n_i, n_j) = 1$ if $i \neq j$, the sequence $\{p_j\}_{j \in N}$ consists of distinct terms. Observe that $n_j \geq p_j^2$ for all positive integers j. Therefore, $\sum_{j=1}^{\infty} \frac{1}{n_j} \leq \sum_{j=1}^{\infty} \frac{1}{p_j^2} < \sum_{j=1}^{\infty} \frac{1}{j^2} = \frac{\pi^2}{6} < \infty$.

The problem was solved by:

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