PROBLEM OF THE WEEK

Solution of Problem No. 11 (Fall 2011 Series)

Problem: There are n men in a warehouse, with no three in a straight line, and so that the distances between pairs of men are distinct.

Each man has a loaded pistol. At a signal, each shoots the man closest to him. Show that if n is odd, then at least one man remains alive. Show also that if n is even, then it is possible that every man dies.

Solution: (by Kilian Cooley, Junior, Math & AAE, Purdue University)

Since the distances between pairs of men are distinct, there is a single minimum distance. The two men A and B separated by this distance must shoot each other, since no other man is closer to either of them. If one of the other $n-2$ men shoots either A or B, then A or B is shot twice. Since a total of n shots are fired, this implies that at least one man survives by the pigeonhole principle. If, however, none of the $n-2$ men shoot A or B, then A and B can be removed from consideration without affecting the parity of the number of men or the outcome of the shootout among the $n-2$. Considering only the men other than A and B, there must again be a single least distance and thus two men who shoot each other, and if either of them is shot twice then one man survives and if not then they may also be ignored. Repeat this process until it is found that one man survives or until all the men have been eliminated. Suppose n is odd and the cases where $2, 4, 6, \ldots, n-3$ men are ignored all fail to show the survival of at least one man. Of the remaining three, two must shoot each other and the third must shoot one of those two, leaving the third man alive. Therefore if n is odd, at least one man survives.

If n is even, then it may be that there exist $n/2$ pairs of men who shoot each other, in which case every man dies. Such a situation could be constructed by considering the set D_k of distances between the two men is less than the minimum of D_k and so that the minimum distance between either of the two and any of the preceding $2k$ men exceeds the maximum of D_k. Since there are finitely many men previously placed, it is also possible to place the $(k+1)$th pair so that the distances between men are all distinct and that no three lie on a line.

(**Remark:** This problem was adapted from a problem in an old Soviet math contest to give a nod to Tarantino's movie *Reservoir Dogs*.)

The problem was also solved by:

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