

PROBLEM OF THE WEEK
Solution of Problem No. 3 (Fall 2011 Series)

Problem: There are nine points in the interior of a cube of side 1. Show that at least two of the points are less than $\sqrt{3}/2$ apart.

Can $\sqrt{3}/2$ be replaced by a smaller number?

Solution: (by Bennett Marsh, Freshman, Purdue University)

Divide the cube into 8 smaller cubes, each of side $1/2$. Since the diagonal of each of these cubes is $\sqrt{3}/2$, two points in any cube must be within that distance of each other. We can place the first 8 points in one box each, but the ninth point must fall into a box that already has a point, and thus must be within $\sqrt{3}/2$ of at least one other point.

The number $\sqrt{3}/2$ cannot be reduced any further, because it is possible to place the points so that the minimum distance between any two is arbitrarily close to $\sqrt{3}/2$. Place one point in the exact center of the cube, and let the other 8 points get arbitrarily close to the 8 vertices. Since the diagonal of the cube is $\sqrt{3}$, the distance from the center point to the other points gets arbitrarily close to $\sqrt{3}/2$.

The problem was also solved by:

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