

PROBLEM OF THE WEEK  
Solution of Problem No. 2 (Spring 2005 Series)

**Problem:** Determine the smallest integer that is a square and whose decimal representation starts with 2005. Calculators may be used but solutions by computers will not be accepted.

**Solution** (by Georges Ghosn, Quebec, edited by the Panel)

The smallest integer  $N = x^2$  must satisfy:

$$2005 \times 10^n \leq x^2 < 2006 \times 10^n$$

where  $n + 4$  is the number of digits in the decimal representation of  $N$ .

We examine 2 cases “ $n$  even” and “ $n$  odd”

For  $n = 2p$  (even)

$$2005 \times 10^{2p} \leq x^2 < 2006 \times 10^{2p} \Rightarrow 44,7772 \dots \times 10^p \leq x < 44,7883 \dots \times 10^p.$$

The smallest value of  $p$  for which  $x$  exists is  $p = 2 \Rightarrow x = 4478$  and  $N = x^2 = 20052484$ .

For  $n = 2p + 1$  (odd)

$$20050 \times 10^{2p} \leq x^2 < 20060 \times 10^{2p} \Rightarrow 141,598 \dots \times 10^p \leq x < 141,633 \dots \times 10^p.$$

The smallest value of  $p$  for which  $x$  exists is  $p = 1 \Rightarrow x = 1416$  and  $N = x^2 = 2005056$ .

Finally the smallest integer is 2005056.

Also solved by:

Undergraduates: Jason Anema (Sr. MA), Alan Bernstein, Chris Giznopoulous (Fr.), Justin Woo (Jr. ECE)

Graduates: Tom Engelsman, Ashish Rao (ECE)

Others: John Hunckler (W. Lafayette), Bhilahari Jeevanesan (Germany), Carol Kupier (Instructor, Purdue), Jeff Ledford (Gainesville, GA), M. Rappaport (Worcester Yeshiva Acad.), S.Sanjiv (ECE, Waterloo), Jack Spade (U. of Minnesota), Gabriel Vrinceanu (Bucharest)