PROBLEM OF THE WEEK

Solution of Problem No. 14 (Fall 2000 Series)

Problem: Consider the equations $N = x^3(3x+1) = y^2(y+1)^3$, where x, y are relatively prime positive integers. Show that there is only one possible value for N. Find it.

Solution (by Steven Landy, Fac. Physics at IUPUI)

Since $(x, y) = 1$ it follows that $x^3|(y+1)^3$, hence $x|(y+1)$, $x \leq y+1$. Similarly $y^2/(3x+1)$, so $y^2 \leq 3x+1$. Combining the inequalities gives $x^2 - 5x = 0$. As x is positive, only $x = 1, 2, 3, 4, 5$ are possible. Trying these values in $x^3(3x+1) = y^2(y+1)^3$, we find that only $x = 5$, $y = 4$ work, so $N = 4^2 \cdot 5^3 = 2000$ is the only solution.

Also solved by:

Undergraduates: Stevie Schraudner (Jr. CS)

Graduates: Vikram Buddhi (MA), Gajath Gunatillake (MA), Yi-Ru Huang (Stat), Chris Lomont (MA)

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Two incorrect solutions were received.