THE 3RD SYMPOSIUM ON ANALYSIS AND PDES

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ISOPERIMETRIC INEQUALITIES RELATED TO MASS AND ENERGY IN GENERAL RELATIVITY

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Asymptotically flat 3-manifolds of non-negative scalar curvature arise as 3-dimensional spacelike (maximal) slices of Lorentzian 4-manifolds modelling isolated gravitating systems such as stars, binaries, galaxies and black holes. Traditionally the total mass of an isolated system is defined by a flux integral near infinity involving first derivatives of the asymptotically flat metric. The course explains how this concept of mass can be replaced both locally and globally by by a more geometric concept describing the mass as a defect in the classical isoperimetric inequality. Using geometric evolution equations such as mean curvature flow and inverse mean curvature flow we show that the isoperimetric concept is compatible with the more classical concepts of ADM-mass and Hawking-mass and derive positive lower bounds related to the positive mass theorem and the Penrose inequality.

Lectures:

1. Concepts for mass and energy of isolated systems in General Relativity.
2. Geometric evolution equations and monotonicity formulae.
3. Isoperimetric inequalities via geometric evolution equations and the isoperimetric concept of mass.
4. Positive mass theorems for the isoperimetric mass and a concept of quasilocal isoperimetric mass.