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A FREE-BOUNDARY PROBLEM FOR THE EVOLUTION p-LAPLACIAN EQUATION WITH HEAT COMBUSTION BOUNDARY CONDITION

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We consider the following free-boundary problem: given a non-negative function u_0 defined in $\mathbb{R}^n$, whose positivity set is a bounded domain Ω_0, find a domain $\Omega \subset \mathbb{R}^n \times [0, T]$ for some positive T and a function $u(x, t)$ such that

$$\begin{cases} u_t = \Delta_p u & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u = 0 \text{ and } |Du| = 1 & \text{on } \partial\Omega \cap \{0 < t < T\} \\ u(\cdot, 0) = u_0 & \text{in } \Omega_0 \end{cases} \tag{1}$$

The operator

$$\Delta_p u = \operatorname{div}\left(|Du|^{p-2}\, Du\right)$$

is known as the p-Laplacian.

This problem in the case $p = 2$ has been studied by Caffarelli and Vazquez. In this work, I will prove the following result in the case $p > 2$:

Theorem. *Let Ω_0 be a bounded convex domain whose boundary is $C^{1+\alpha}$. The function $u_0(x)$ is concave and in $C^{1+\alpha}(\Omega_0)$. Then there is a solution (u, Ω) of the problem* (1) *which exists up to a vanishing time T.*

Moreover, the vanishing time T is finite, the free-boundary $\partial\Omega_t$ is in C^∞ for positive t and this solution is unique.

References

[1] L.A.Caffarelli and J.L.Vazquez, *A free boundary problem for the heat equation arising in flame propagation*, Trans. Amer. Math. Soc.**347** (1995), 411-441.