6th Symposium on Analysis and PDEs

Purdue University, June 1–4, 2015

ABSTRACTS

Principal Lecturers

David Jerison, Massachusetts Institute of Technology *3 lectures: June 2–4, 9:00–9:50am*

MINICOURSE: ***Free Boundaries and Minimal Surfaces.***

Free boundaries arise as the interface between materials in which the materials retain some energy. In contrast, the interface represented by a minimal surface lives in an ambient space that is empty. Despite this difference between these two types of interfaces, there are deep connections between them. In a ground-breaking paper in 1980, Alt and Caffarelli showed that strategies and tools from the theory of minimal surfaces yield fundamental regularity theorems for free boundaries. The field has exploded since then.

In these lectures, we will start by explaining the wide family of energy functionals that can be treated by free boundary methods developed in that last 35 years. We will then focus on recent progress describing minimizers in higher dimensions and higher critical points in dimension 2. These are contexts in which free boundary problems return the favor by pointing to new insights into minimal surfaces.

Fang-Hua Lin, Courant Institute, New York University *3 lectures: June 1–3, 2:30–3:20pm*

MINICOURSE: ***Extremum Problems for Elliptic Eigenvalues.***

In these lectures, we are going to discuss some simple but fundamental, extremum problems for Laplacian eigenvalues. They are interesting from both theory and applications. The goal of the lectures is to explain some relevant classical works and some recent thoughts on these problems.

Invited Speakers

Mark Allen, University of Texas at Austin *June 1, 11:30–12:20pm*

Parabolic Problems with a Fractional Time Derivative.

In this talk we will begin by discussing the different notions of a fractional-time derivative and their applications in physics. We will then focus on some specific parabolic problems involving fractional time derivatives as well as fractional spatial derivatives. These fractional parabolic problems have particular relevance in modeling plasma transport problems. We will discuss existence, uniqueness, and regularity for these problems. This is joint work with L. Caffarelli and A. Vasseur.

Daniela De Silva, Barnard College, Columbia University *June 1, 9:00–9:50am*

Higher Order Boundary Harnack Inequalities and Applications to Free Boundary Problems.

We present some higher order Boundary Harnack inequalities and provide applications to obtain higher regularity in obstacle-type free boundary problems. In particular, we describe the boundary behavior of harmonic functions in so-called "slit domains" and develop the appropriate Schauder theory. This is a joint work with O. Savin.

Ailana Fraser, University of British Columbia *June 2, 10:00–10:50am*

Minimal Surfaces and an Extremal Eigenvalue Problem.

Finding sharp eigenvalue bounds and characterizing the extremals is a basic problem in geometric analysis. We will describe the structure of metrics which are obtained by maximizing the first eigenvalue of the Dirichlet-to-Neumann map over all metrics on a surface with boundary. It turns out that the extremals are related to minimal surfaces in the ball with a natural boundary condition, and in some cases it is possible to use minimal surface theory to characterize the extremal metrics and obtain sharp eigenvalue bounds. This is joint work with R. Schoen.

Nikola Kamburov, University of Arizona *June 3, 10:00–10:50am*

The Space of One-Phase Free Boundary Solutions in the Plane.

In joint work with David Jerison we study the compactness of the space of solutions to the one-phase free boundary problem in the disk that have simply-connected positive phase. In this talk I will present the classification of blow-up limits (namely, the limit is either the one-plane, the two-plane or the double hairpin solution discovered by Hauswirth, Helein and Pacard) and discuss how the result informs the characterization of the local structure of the free boundary.

Yuan Lou, Ohio State University *June 2, 11:30–12:20pm*

Asymptotic Behavior of the Smallest Eigenvalue of an Elliptic Operator and its Applications to Evolution of Dispersal.

We investigate the effects of diffusion and drift on the smallest eigenvalue of an elliptic operator with zero Neumann boundary condition. Various asymptotic behaviors of the smallest eigenvalue, as diffusion and drift rates approach zero or infinity, are derived. As an application, these qualitative results yield some insight into the evolution of dispersal in heterogeneous environments.

Luis Silvestre, University of Chicago *June 1, 10:00–10:50am*

$C^{1,\alpha}$ regularity for the parabolic homogeneous p-Laplacian equation.

It is well known that p-harmonic functions are $C^{1,\alpha}$ regular, for some $\alpha > 0$. The classical proofs of this fact use variational methods. In a recent work, Peres and Sheffield construct p-Harmonic functions from the value of a stochastic game. This construction also leads to a parabolic versions of the problem. However, the parabolic equation derived from the stochastic game is not the classical parabolic p-Laplace equation, but a homogeneous of degree one version. This equation is not in divergence form and variational methods are inapplicable. We prove that solutions to this equation are also $C^{1,\alpha}$ regular in space. This is joint work with Tianling Jin.

Hung Tran, University of Chicago *June 4, 11:30–12:20pm*

Some Inverse Problems in Periodic Homogenization of Hamilton-Jacobi Equations.

We look at the effective Hamiltonian $\overline{H}$ associated with the Hamiltonian $H(p,x) = H(p)+V(x)$ in the periodic homogenization theory. Our central goal is to understand the relation between V and $\overline{H}$. We formulate some inverse problems concerning this relation. Such type of inverse problems are in general very challenging. I will discuss some interesting cases in both convex and nonconvex settings. Joint work with Songting Luo and Yifeng Yu.

Contributed Talks

Eric Baer, Massachusetts Institute of Technology *June 2, 4:30–4:55pm*

Optimal Function Spaces for Continuity of the Hessian Determinant as a Distribution.

In this talk we describe a new class of optimal continuity results for the action of the Hessian determinant on spaces of Besov type into the space of distributions on $\mathbb{R}^N$. In particular, inspired by recent work of Brezis and Nguyen on the distributional Jacobian determinant, we show that the action is continuous on the Besov space of fractional order $B(2-\frac{2}{N}, N)$, and that all continuity results in this scale of Besov spaces are consequences of this result. A key ingredient in the argument is the characterization of $B(2-\frac{2}{N}, N)$ as the space of traces of functions in the Sobolev space $W^{2,N}(\mathbb{R}^{N+2})$ on the subspace $\mathbb{R}^N$ of codimension 2. The most delicate and elaborate part of the analysis is the construction of a counterexample to continuity in $B(2-\frac{2}{N}, p)$ with $p > N$. This is joint work with D. Jerison.

Soledad Benguria, University of Wisconsin-Madison *June 3, 4:00–4:25pm*

The Brezis-Nirenberg Problem on $\mathbb{S}^n$, in Spaces of Fractional Dimension.

We consider the nonlinear eigenvalue problem,

$$-\Delta_{\mathbb{S}^n} u = \lambda u + |u|^{4/(n-2)} u,$$

with $u \in H_0^1(\Omega)$, where Ω is a geodesic ball in $\mathbb{S}^n$ contained in a hemisphere. In dimension 3, Bandle and Benguria proved that this problem has a unique positive solution if and only if

$$\frac{\pi^2 - 4\theta_1^2}{4\theta_1^2} < \lambda < \frac{\pi^2 - \theta_1^2}{\theta_1^2}$$

where θ_1 is the geodesic radius of the ball. For positive radial solutions of this problem one is led to an ODE that still makes sense when n is a real number rather than a natural number. We consider precisely that problem with $2 < n < 4$. Our main result is that in this case one has a positive solution if and only if λ is such that

$$\frac{1}{4}[(2\ell_2+1)^2 - (n-1)^2] < \lambda < 1[(2\ell_1+1)^2 - (n-1)^2]$$

where ℓ_1 (respectively ℓ_2) is the first positive value of ℓ for which the associated Legendre function $P_\ell^{(2-n)/2}(\cos\theta_1)$ (respectively $P_\ell^{(n-2)/2}(\cos\theta_1)$) vanishes. Joint work with Rafael D. Benguria (arXiv:1503.06347)

Matias Delgadino, University of Maryland-College Park *June 1, 5:00–5:25pm*

The Relationship Between the Obstacle Problem and Minimizers of the Interaction Energy.

The repulsion strength at the origin for repulsive/attractive potentials determines the minimal regularity of local minimizers of the interaction energy. If the repulsion is like Newtonian or more singular than Newtonian (but still locally integrable), then the local minimizers must be locally bounded densities (and even continuous for more singular than Newtonian repulsion). This can be achieved by first showing that the potential function associated to a local minimizer solves an obstacle problem and then by using classical regularity results for such problems.

Max Engelstein, University of Chicago *June 3, 11:30–11:55am*

A Free Boundary Problem for the Parabolic Poisson Kernel.

We study parabolic chord arc domains, introduced by Hofmann, Lewis and Nyström (*Duke*, '04), and prove a free boundary regularity result below the continuous threshold. More precisely, we show that a Reifenberg flat, parabolic chord arc domain whose Poisson kernel has logarithm in VMO must in fact be a vanishing chord arc domain (i.e. satisfies a vanishing Carleson measure condition). This generalizes, to the parabolic setting, a result of Kenig and Toro (*Ann. ENS*, '03) and answers in the affirmative a question left open in the aforementioned paper of Hofmann et al.

Rohit Jain, University of Texas at Austin *June 1, 3:30–3:55pm*

Randomly Homogenized Boundary Obstacle Problem.

The aim of the talk is to study a problem motivated by a mathematical model of semipermeable membranes. The basic question we aim to address is the regularity property for the homogenized solution to a boundary obstacle problem in a perforated domain, where the the holes are periodically distributed and have random shape and size. We obtain uniform $C^{1,1/2}$ estimates up to the boundary for the solution independent of the penalty constant λ which appears in the homogenized equation. This allows us to prove uniform convergence to the regular $C^{1,1/2}$ solution of the Thin Obstacle Problem.

Alessia Kogoj, University of Bologna *June 1, 4:30–4:55pm*

Weighted L^p-Liouville Theorems for Hypoelliptic Partial Differential Operators on Lie Groups.

We show several weighted L^p-Liouville-type theorems for second order hypoelliptic partial differential operators on Lie groups in $\mathbb{R}^N$. We provide examples of operators to which our results apply (e.g. heat operators on Carnot groups and Kolmogorov-Fokker-Planck operators) and an application to the uniqueness for the Cauchy problem for the evolution operators $\mathcal{L} - \partial_t$. The results presented are obtained in collaboration with A. Bonfiglioli and E. Lanconelli.

Lyudmila Korobenko, McMaster University *June 3, 12:00–12:25pm*

Boundedness and Continuity of Solutions to Infinitely Degenerate Elliptic Equations via Sobolev Inequalities for Associated Metrics.

The talk is concerned with regularity of weak solutions to second order infinitely degenerate elliptic equations. It is known that regularity of weak solutions can be studied by studying properties of certain metric spaces associated to the operator, namely subunit metric spaces. The problem arising in the infinitely degenerate case is that the measures of subunit balls are non doubling. As a consequence many classical tools such as Sobolev-type inequalities become unavailable. We show that in certain cases a weaker version of Sobolev inequality can be established which allows to perform a "standard" Moser iteration scheme to obtain boundedness and continuity of weak solutions.

Dennis Kriventsov, University of Texas at Austin *June 4, 10:00–10:25am*

A Free Boundary Problem Related to Thermal Insulation.

We consider a variational problem for domains coming from the task of finding an optimal thermal insulator. Let $\Omega \subseteq \mathbb{R}^n$ be a (nice) fixed set, and minimize

$$F(A, u) = \int_A |\nabla u|^2 d\mathcal{L}^n + h \int_{\partial A} u^2 d\mathcal{H}^{n-1} + C_0 \mathcal{L}^n(A \setminus \Omega)$$

over all sets A containing Ω and having smooth boundary, and all smooth functions $u \in C^1(A)$ with $u \equiv 1$ on Ω. Here h and C_0 are fixed, positive parameters. This may be thought of as a variational free boundary problem, with the unusual characteristic that along the boundary of the minimal set, ∂A, the harmonic function u satisfies a Robin condition, not the typical Dirichlet condition. We will briefly explain how this problem arises, discuss how an appropriately relaxed version of our functional admits minimizers, and then describe the regularity properties of these minimizers. This is based on joint work with Luis Caffarelli.

Ryan Murray, Carnegie Mellon University *June 2, 5:00–5:25pm*

Second-Order Gamma Limit for the Cahn-Hilliard Functional.

The Cahn–Hilliard functional models phase transitions in a variety of physical settings. This talk will discuss the resolution of a long standing open problem, namely, the asymptotic development of order 2 by Γ-convergence of the mass-constrained Cahn–Hilliard functional. This is achieved by introducing a novel rearrangement technique, which works without Dirichlet boundary conditions.

Robin Neumayer, University of Texas at Austin *June 2, 3:30–3:55pm*

A Strong Form of the Quantitative Wulff Inequality.

For a set E that almost minimizes perimeter among sets of the same volume, quantitative isoperimetric inequalities measure how "close" this set is to the unique perimeter minimizer. A recent paper of Fusco and Julin gives quantitative control of the oscillation of the boundary of such a set. In this talk, we will generalize this result for the anisotropic case, where perimeter is weighted with respect to some fixed convex set K.

Veronica Quitalo, Purdue University *June 4, 10:30–10:55am*

On a Long Range Segregation Model.

In this talk we will present the main results about a free boundary problem arising from a long range segregation process. In this model, the growth of a population u_i at x is inhibited by the populations u_j in a full area surrounding x. Heuristically, this will force the populations to stay at distance 1 from each other in the limit configuration. These models have some similarity with the Lasry-Lions model of price formation, where selling and buying prices are separated by a gap due to transaction cost.

Drew Swartz, Purdue University *June 3, 3:30–3:55pm*

Dynamics of a Second Order Gradient Model for Phase Transitions.

In 2000 Fonseca and Mantegazza introduced a second order gradient theory for phase transitions. The model is similar in spirit to the Ginzburg-Landau model, which is a first order model. In their work, Fonseca and Mantegazza showed that their energy Γ-converges to a perimeter functional. In this talk we will discuss the gradient flow dynamics for the Fonseca-Mantegazza energy. The corresponding evolution equation is fourth order, thus creating some interesting difficulties in its analysis. We analyze properties of the optimal transition profile through a combination of analytical and numerical techniques. Then in the radially symmetric setting, we use this to demonstrate that the gradient flow dynamics converges to motion by mean curvature. This is joint work with Prof. Aaron Yip.

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