

MA366 MIDTERM EXAM 1 – PRACTICE PROBLEMS

1. If  $y' + (1 + \frac{1}{t})y = \frac{1}{t}$  and  $y(1) = 0$ , then  $y(\ln 2) = ?$
2. What is the largest open interval for which a unique solution of the initial value problem

$$ty' + \frac{1}{t+1}y = \frac{t-2}{t-3}, \quad y(1) = 0$$

is guaranteed?

3. Consider the autonomous differential equation

$$\frac{dy}{dt} = -\frac{1}{10}(y-1)(y-4)^2.$$

Classify the stability of each equilibrium solution.

4. Determine whether

$$x + 2y + (2x + y)\frac{dy}{dx} = 0$$

is separable, homogeneous, linear and/or exact.

5. An explicit solution of  $y' = y^2 - 1$  is?
6. If  $y' = y^3$  and  $y(0) = 1$ , then  $y(-1) = ?$
7. If  $(2x^2 + y^2)dx - xydy = 0$  and  $y(1) = 2$ , then  $y(e^3) = ?$
8. An implicit solution of

$$y^2 + 1 + (2xy + 1)\frac{dy}{dx} = 0$$

is?

9. If  $y'$  is proportional to  $y$ ,  $y(0) = 2$  and  $y(1) = 8$ , for what value of  $t$  does  $y(t) = 20$ ?

10. The general solution of

$$y'' - 4y' + 4y = 0$$

is?

11. The general solution of

$$y''' + 4y'' + 5y' = 0$$

is?

12. A particular solution,  $y_p$ , of

$$y'' - 4y' + 3y = 2t + e^t$$

is?

13. If  $y'' + 5y' + 6y = 24e^t$ ,  $y(0) = 0$ ,  $y'(0) = 0$ , then  $y(1) = ?$

14. The differential equation

$$y'' - \frac{2}{t}y' + \frac{2}{t^2}y = 0$$

has solutions  $y_1(t) = t$  and  $y_2(t) = t^2$ . If

$$y'' - \frac{2}{t}y' + \frac{2}{t^2}y = 2; \quad y(1) = 0, \quad y'(1) = 0$$

then  $y(2) = ?$

15. A tank initially contains 40 ounces of salt mixed in 100 gallons of water. A solution containing 4 oz of salt per gallon is ten pumped into the tank at the rate of 5 gal/min. The stirred mixture flows out of the tank at the same rate. How much salt is in the tank after 20 minutes?