MA366 MIDTERM EXAM 2 – PRACTICE PROBLEMS

1. The general solution of

$$y''' + 4y'' + 5y' = 0$$

is?

2. Find the solution of the initial value problem

$$y^{(4)} + 2y'' + y = 3t + 4; \quad y(0) = y'(0) = 0, \quad y''(0) = y'''(0) = 1$$

by using the method of undetermined coefficients.

3. An object weighting 8 pounds attached to a spring will stretch it 6 inches beyond its natural length. There is a damping force with a damping constant $c = 6$ lbs-sec/ft and there is no external force. If at $t = 0$ the object is pulled 2 feet below equilibrium and then released, the initial value problem describing the vertical displacement $x(t)$ becomes?

4. A spring-mass system is governed by the initial value problem

$$x'' + 4x' + 4x = 4\cos \omega t$$
$$x(0) = 9, \quad x'(0) = -2.$$

For what value(s) of ω will resonance occur?

5. $\mathcal{L}\{e^t(1 + \cos 2t)\} = ?$

6. Find the Laplace transform of

$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & 1 \le t < \infty \end{cases} .$$

7. Solve

$$y'' + 3y' + 2y = 4u_1(t)$$
$$y(0) = 0, \quad y'(0) = 1.$$

8. Find the solution of the initial value problem

$$y'' + y = \delta(t - \pi)$$
$$y(0) = 0, \quad y'(0) = 1.$$

9. The inverse Laplace transform of

$$F(s) = \frac{se^{-s}}{s^2 + 2s + 5}$$

is?

10. $\mathcal{L}\left\{\int_0^t \sin 2(t - \tau)\cos(3\tau)d\tau\right\} = ?$

11. Using the method of undetermined coefficients, determine the *form* of the particular solution $y_p(t)$ to the differential equation

$$y''' - y'' - y' + y = 2t + 3e^t.$$

A. $y_p(t) = 2t + at^2e^t + bte^{-t}$
B. $y_p(t) = at + b + ct^2e^t$
C. $y_p(t) = at + be^t + cte^t + dt^2 + e^t$
D. $y_p(t) = a + bt^2e^t$

12. If the characteristic equation has roots $r = 1, -1+2i, -1-2i$, with multiplicities $2, 1, 1$ respectively, then a fundamental solution set is given by
A. $\{e^t, e^{-t}\cos 2t, e^{-t}\sin 2t\}$
B. $\{e^t, te^t, e^{-t}\cos 2t, e^{-t}\sin 2t\}$
C. $\{e^t, te^t, e^{-2t}\cos t, e^{-2t}\sin t\}$
D. $\{e^{-t}, te^{-t}, e^{-t}\cos 2t, e^{-t}\sin 2t\}$

13. Given $f(t) = \begin{cases} 1 & \text{if } 0 \le t < 1 \\ t & \text{if } 1 \le t \end{cases}$, determine $F(s) = \mathcal{L}\{f\}$.

A. $\dfrac{1}{s^2}$
B. $\dfrac{s + e^{-s}}{s^2}$
C. $\dfrac{(s+1)e^{-s}}{s^2}$
D. $\dfrac{1}{s}$

14. Let $y(t)$ be the solution of the initial value problem

$$y'' + 2y' + 3y = \delta(t-1) + u_2(t), \quad y(0) = 0, \quad y'(0) = 1.$$

Compute the Laplace transform of $y(t)$.
A. $Y(s) = \dfrac{1 + e^{-s}}{s^2 + 2s + 3} + \dfrac{e^{-2s}}{s(s^2 + 2s + 3)}$
B. $Y(s) = \dfrac{e^{-s} - 2}{s^2 + 2s} + \dfrac{e^{-2s}}{s(s^2 + 2s)}$
C. $Y(s) = \dfrac{2 + s + e^{-s}}{s^2 + 2s + 3} + \dfrac{e^{-2s}}{s(s^2 + 2s + 3)}$
D. $Y(s) = \dfrac{1 + e^{-2s}}{s^2 + 2s + 3} + \dfrac{e^{-s}}{s(s^2 + 2s + 3)}$

15. Find the inverse Laplace transform of $F(s) = \dfrac{1 - e^{-\pi s}}{s(s^2+1)}$.
A. $1 - \cos t - u_\pi(t)(1 + \cos t)$
B. $(1 - e^{-\pi t})(1 - \cos t)$
C. $1 - \cos(t) - \delta(t - \pi)$
D. $1 - \cos t - u_\pi(t)(1 - \cos t)$

16. Let $f(t) = \int_0^t (t-\tau)^3 \cos 2\tau \, d\tau$. Find the Laplace transform of $f(t)$.

A. $\dfrac{6}{s^4} + \dfrac{s}{s^2+4}$

B. $\dfrac{12}{s^4(s^2+4)}$

C. $\dfrac{6}{s^3(s^2+4)}$

D. $\dfrac{6(s-2)}{s^4(s^2+4)}$

17. Find the inverse Laplace transform of $\dfrac{2s+1}{s^2-3s-4}$

A. $(7/5)e^{-4t} + (3/5)e^{t}$

B. $(9/5)e^{4t} + (1/5)e^{-t}$

C. $9e^{4t} + e^{-t}$

D. $\dfrac{9/5}{t-4} + \dfrac{1/5}{t+1}$

18. Solve the initial value problem

$$y'' - 4y = \delta(t-1), \quad y(0) = 0, \quad y'(0) = 0$$

A. $y(t) = u_2(t)\sinh(t-2)$

B. $y(t) = \dfrac{1}{2}u_1(t)\sinh 2t$

C. $y(t) = \dfrac{1}{2}u_1(t)\sinh(2t-1)$

D. $y(t) = \dfrac{1}{2}u_1(t)\sinh(2t-2)$