MA490F FOURIER ANALYSIS

PURDUE UNIVERSITY, SPRING 2008

MIDTERM EXAM 2

1. Prove that if f is continuous, of moderate decrease and $\int_{-\infty}^{\infty} f(y)e^{-y^2}e^{2xy}dy = 0$ for all $x \in \mathbb{R}$, then $f = 0$.

[Hint: Consider $f * e^{-x^2}$]

2. Show that for any $a \neq 0$ and σ with $0 < \sigma < 1$, the sequence $\langle an^\sigma \rangle$ is equidistributed in $[0, 1)$.

[Hint: Prove that $\sum_{n=1}^{N} e^{2\pi i b n^\sigma} = O(N^\sigma) + O(N^{1-\sigma})$ if $b \neq 0$.] In fact, note the following

$$\sum_{n=1}^{N} e^{2\pi i b n^\sigma} - \int_1^N e^{2\pi i b x^\sigma} dx = O\left(\sum_{n=1}^{N} n^{-1+\sigma}\right).$$

3. Define the theta function $\vartheta(s)$ for $s > 0$ by

$$\vartheta(s) = \sum_{n=-\infty}^{\infty} e^{-\pi n^2 s}.$$

Prove the identity

$$\frac{1}{\sqrt{s}}\vartheta\left(\frac{1}{s}\right) = \vartheta(s), \qquad s > 0.$$