72nd Midwest PDE Seminar

Purdue University, November 16–17, 2013

Titles & Abstracts

Invited Speakers

Alessio Figalli, *University of Texas at Austin* — Sat, Nov 16, 9:00–9:50, MATH 175

Partial regularity for Monge-Ampère type equations

Abstract. Monge-Ampère type equations arise in several problems from analysis and geometry, and understanding their regularity is an important question. In particular, this kind of equations arises in the regularity theory of optimal transport maps.

In the 90's Caffarelli developed a regularity theory on $\mathbb{R}^n$ for the classical Monge-Ampère equation, which was then extended by Ma-Trudinger-Wang and Loeper to a more general class of equations which satisfy a suitable structural condition. Unfortunately, this condition is very restrictive and it is satisfied only in very particular cases. Hence the need to develop a partial regularity theory: is it true that solutions are always smooth outside a "small" singular set? The aim of this talk is first to review the "classical" regularity theory, and then to describe some recent results about partial regularity.

Konstantina Trivisa, *University of Maryland* — Sat, Nov 16, 10:00–10:50, MATH 175

On kinetic models for the collective self-organization of agents

Abstract. A class of kinetic models is presented for the collective self-organization of agents. Results on the global existence of weak solutions as well as a hydrodynamic limit will be discussed. The main tools employed in the analysis are the velocity averaging lemma and the relative entropy method. This is joint work with T. Karper and A. Mellet.

Shu-Ming Sun, *Virginia Tech* — Sat, Nov 16, 2:00–2:50, MATH 175

Solitary water waves and their stability

Abstract. The talk will discuss recent development on the existence and stability of two- and three-dimensional solitary waves on the water of finite depth with or without surface tension using the exact governing equations (called Euler equations). It will be shown that when the non-dimensional wave-speed and surface tension are in various regions, the Euler equations possess several different kinds of two- or three-dimensional solitary-wave solutions. Moreover, various stability results for these waves will be addressed, such as transverse instability, spectral stability, asymptotic linear stability or conditional stability.

Charles Smart, *MIT* Sat, Nov 16, 3:00–3:50, MATH 175

Quantitative stochastic homogenization of non-divergence form elliptic equations

Abstract. I will discuss joint work with Scott Armstrong on a new method for studying stochastic homogenization of elliptic equations in nondivergence form. Our main application is an algebraic error estimate in the finite range of dependence case which is new even for linear equations. Our proofs rely on a new geometric quantity which we control using the regularity theory for the Monge-Ampere equation.

Robert Jerrard, *University of Toronto* Sun, Nov 17, 9:00–9:50, MATH 175

Existence and uniqueness of minimizers of general least gradient problems

Abstract. Motivated by problems arising in conductivity imaging, we prove existence, uniqueness, and comparison theorems for minimizers of the Dirichlet problem for the functional

$$I[u] := \int_\Omega a(x)|Du|dx,$$

as well as more general minimization problems of the same type. In particular, we find geometric conditions on the domain Ω and regularity hypotheses on the coefficient function a that are sufficient to guarantee uniqueness of minimizers, and we construct examples to show that these conditions are sharp in a certain sense. This is joint work with Amir Moradifam and Adrian Nachman.

Shankar Venakataramani, *University of Arizona* Sun, Nov 17, 10:00–10:50, MATH 175

Singular interfaces in two dimensional free boundary problems

Abstract. I will talk about some questions that arise in attempting to apply conformal mapping techniques to free boundary problems in 2D electrostatics and fluid flows, especially in describing multi-scale or “nearly singular” interfaces. This is ongoing work in collaboration with Stuart Kent.

Vladimir Sverak, *University of Minnesota* Sun, Nov 17, 11:30–12:20, MATH 175

Invariant measures and long-time behavior for some Hamiltonian PDEs

Abstract. Long-time behavior of finite-dimensional Hamiltonian systems is a classical subject known for its difficulty. Infinite dimension brings new phenomena which can in some sense simplify the behavior. Rigorous results fully capturing these effects are hard to obtain, but interesting theorems can be proved even at a relatively “soft” level. We will discuss some of them.

Contributed Speakers

Ko-Shin Chen, *Indiana University* Sat, Nov 16, 11:30–11:50, REC 121

Dynamics of Ginzburg-Landau and Gross-Pitaevskii vortices on manifolds

Abstract. In this talk we consider the dissipative heat flow and conservative Gross-Pitaevskii dynamics associated with the Ginzburg-Landau energy posed on a 2-manifold. We will show that in the $\varepsilon \to 0$, the vortices of the solution to these two problems evolve according to the gradient flow and Hamiltonian point-vortex flow respectively, associated with the renormalized energy. For the heat flow on a sphere, we will also present an annihilation result for the limiting system of ODE's.

Mimi Dai, *University of Illinois, Chicago* Sat, Nov 16, 11:30–11:50, REC 122

Stability of solutions to the dissipative quasi-geostrophic equations

Abstract. We consider the steady-state Surface Quasi-Geostrophic equation in the whole space $\mathbb{R}^2$ driven by a forcing function f. The class of source function f under certain assumptions yield the existence of at least one solution with finite energy (finite L^2 norm). These solutions are unique among all solutions with finite energy. The constructed solutions are also shown to be stable in the following sense: If Θ is such a solution then any viscous, incompressible flow in the whole space, driven by f and starting with finite energy, will return to Θ.

Guanying Peng, *Purdue University* Sat, Nov 16, 11:55–12:15, REC 121

Properties of minimizers of the Lawrence-Doniach energy with perpendicular magnetic field

Abstract. We analyze minimizers of the Lawrence-Doniach energy for layered superconductors occupying a bounded generalized cylinder. For an applied magnetic field in the intermediate regime that is perpendicular to the layers, we prove an asymptotic formula for the minimum Lawrence-Doniach energy as the reciprocal of the Ginzburg-Landau parameter and the interlayer distance tend to zero. Under an appropriate assumption on the relationship between these two parameters, we establish comparison results between the minimum Lawrence-Doniach energy and the minimum 3D anisotropic Ginzburg-Landau energy.

Changhui Tan, *University of Maryland* Sat, Nov 16, 11:55–12:15, REC 122

Critical thresholds on compressible Eulerian dynamics with nonlocal alignment

Abstract. We consider compressible Eulerian dynamics with a nonlocal interaction term. The system arises from hydrodynamic flocking models which characterize the phenomenon that individuals emerge to the same direction under simple interactions. We show that any strong solution of the system has the flocking property. In this talk, I will discuss when the system has a global strong solution. Our main result says that there are critical thresholds for the existence of global strong solution, and the thresholds depend only on the initial configuration. A subcritical initial data will lead to strong solution and converges to a flock. On the other hand, a supercritical initial data will blowup in finite time. This is a joint work with Eitan Tadmor.

Renato Ghini Bettiol, *University of Notre Dame* Sat, Nov 16, 4:30–4:50, REC 121

Equivariant bifurcation in 2 geometric variational PDEs

Abstract. I plan to describe how classic variational bifurcation techniques can be adapted to two important geometric variational problems on closed manifolds, namely, finding hypersurfaces of Constant Mean Curvature (CMC) and finding metrics of Constant Scalar Curvature on a given conformal class (Yamabe problem). Both of these problems correspond to solving a nonlinear elliptic PDE on a closed Riemannian manifold, however, our methods only involve the corresponding variational characterizations. Our main results give infinitely many bifurcations for certain families of highly symmetric solutions of these problems, in particular implying the existence of infinitely many new solutions with less symmetry. Concrete geometric examples will be given regarding homogeneous metrics on spheres and homogeneous CMC hypersurfaces in cohomogeneity one manifolds.

Zheng Hao, *Kansas State University* Sat, Nov 16, 4:30–4:50, REC 122

The obstacle problem for elliptic operators in divergence form

Abstract. We describe how we have developed the theory for the divergence form version of the obstacle problem with coefficients in VMO, including existence, uniqueness, nondegeneracy, optimal regularity, and measure stability. In fact, we now even have a stronger version of Caffarelli's regularity theory than Blank and Teka proved, as we can show that the free boundary is a Reifenberg Vanishing set near regular points. This talk is based on joint work with Blank.

Erik Lundberg, *Purdue University* Sat, Nov 16, 4:55–5:15, REC 121

Growth of solutions to the minimal surface equation over unbounded domains

Abstract. Consider a minimal surface which is the graph of a function over an unbounded domain. If the function is positive and vanishes on the boundary of the domain, what can be said about its growth? I will discuss examples and estimates for growth in terms of the geometry of the domain. I will focus on an interesting phase transition in the growth constraints for minimal graphs over domains (a) contained in a halfplane as compared to (b) containing a halfplane. This is joint work with Allen Weitsman.

Nguyen Lam, *Wayne State University* Sat, Nov 16, 4:55–5:15, REC 122

Moser-Trudinger and Adams inequalities and applications

Abstract. In this talk, we will discuss some recent development on sharp subcritical and critical Moser-Trudinger-Adams inequalities on Euclidean spaces and Heisenberg groups. Due to the absence of the symmetrization argument on the Heisenberg group and also on the high order Sobolev spaces, we develop a rearrangement-free method to establish these sharp inequalities. These are joint works with Guozhen Lu.

Murat Akman, *Institut Mittag-Leffler* Sat, Nov 16, 5:20–5:40, REC 121

Hausdorff dimension and p-harmonic measure

Abstract. In the first part of my talk I will discuss Hausdorff dimension of a measure related to a positive weak solution of a certain partial differential equation in a simply connected domain. Our work generalizes work of Lewis and coauthors when the measure is p-harmonic and also for $p = 2$, the well known theorem of Makarov regarding the Hausdorff dimension of harmonic measure relative to a point in a simply connected domain. In the second part of my talk I will present a recent result in the study of Hausdorff dimension of p-harmonic measure for $p \geq n$ when p-harmonic function is defined on an open subset of $\mathbb{R}^n$ and vanishing on a portion of boundary of this open set. Part of this talk is a joint work with John Lewis and Andrew Vogel.

Hussein Awala, *Temple University* Sat, Nov 16, 5:20–5:40, REC 122

On the mixed boundary value problem for the Laplacian in polygonal domains in two dimensions

Abstract. Boundary value problems with mixed Dirichlet and Neumann boundary conditions model a series of physical and engineering phenomena such as conductivity, heat transfer, wave phenomena, electrostatics, metallurgical melting, and stamp problems in elasticity and hydrodynamics. In this talk I will discuss some recent results about the well-posedness of the mixed problem for the Laplacian on curvilinear polygons in two dimensions. The tools employed to obtain these results are a mixture of Calderon-Zygmund theory and Mellin transform techniques.This is joint work with Irina Mitrea and Katharine Ott.