In this assignment, MAPLE will be used to solve a large problem involving the method of undetermined coefficients for a high order linear ODE. First, an example will be worked in MAPLE to give you the tools you will need. Consider the initial value problem
\[ \frac{d^4 y}{dx^4} + 2 \frac{d^3 y}{dx^3} + 3 \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 2 y = \cos(x) \]
with \( y(0)=1, \, y'(0)=2, \, y''(0)=3, \) and \( y'''(0)=4. \) Type in the following commands:

\[ \text{char} \text{poly} := r^4 + 2 \cdot r^3 + 3 \cdot r^2 + 2 \cdot r + 2; \]
\[ \text{factor( char} \text{poly );} \]
\[ \text{solve( char} \text{poly}=0, r ); \]
\[ \text{y} \_\text{homo} := c_1 \cdot \cos(x) + c_2 \cdot \sin(x) + c_3 \cdot e^{-x} \cdot \cos(x) + c_4 \cdot e^{-x} \cdot \sin(x); \]
\[ \text{y} \_\text{p} := x \cdot (a_1 \cdot \cos(x) + a_2 \cdot \sin(x)); \quad \# \text{This is the correct FORM of y} \_\text{p}. \]
\[ \text{diff(y} \_\text{p}, x^{4} ) + 2 \cdot \text{diff(y} \_\text{p}, x^{3} ) + 3 \cdot \text{diff(y} \_\text{p}, x^{2} ) + 2 \cdot \text{diff(y} \_\text{p}, x) + 2 \cdot y \_\text{p}; \]
\[ \text{simplify("); \quad \# \text{simplify previous expression, equate coefficients by hand} \]
\[ \text{solve( \{eqn1,eqn2\} , \{a_1,a_2\});} \]
\[ \text{assign("); \quad \# \text{this makes a}_1=-1/5 \text{ and a}_2=1/10 \text{ from now on} \]
\[ \text{y} \_\text{gen} := y \_\text{p} + y \_\text{homo}; \quad \# \text{y} \_\text{gen is the general solution.} \]
Let's consider the differential equation:

\[ y_{\text{gen}} := x \left( -\frac{1}{5} \cos(x) + \frac{1}{10} \sin(x) \right) + c_1 \cos(x) + c_2 \sin(x) + c_3 e^{-x} \cos(x) + c_4 e^{-x} \sin(x) \]

We solve the initial value problem (IVP) by first substituting \( x = 0 \) into the general solution:

\[ c_1 \cos(0) + c_2 \sin(0) + c_3 e^0 \cos(0) + c_4 e^0 \sin(0) \]

\[ = c_1 + c_3 \]

Then, we evaluate \( y \) and its derivatives at \( x = 0 \) to obtain:

\[ y(0) = c_1 \cos(0) + c_2 \sin(0) + c_3 e^0 \cos(0) + c_4 e^0 \sin(0) \]

\[ = c_1 + c_3 \]

\[ y'(0) = -\frac{1}{5} c_1 - \frac{1}{5} c_2 - c_3 + c_4 \]

\[ = \frac{1}{5} - c_1 - 2 c_4 \]

\[ y''(0) = \frac{3}{5} c_2 + 2 c_3 + 2 c_4 \]

\[ = \frac{3}{5} - c_2 + 2 c_3 + 2 c_4 \]

Next, we solve the system of equations:

\[ \{EQN1, EQN2, EQN3, EQN4\} \]

\[ \{c_1 = \frac{159}{25}, c_2 = \frac{113}{25}, c_3 = \frac{-88}{25}, c_4 = \frac{9}{25}\} \]

Finally, we assign the solution to \( y \):

\[ y := y_{\text{gen}} \]

This is the solution to the IVP. \( y_{\text{gen}} \) is no longer general.
\[ y := x \left( \frac{1}{5} \cos(x) + \frac{1}{10} \sin(x) \right) - \frac{88}{25} \cos(x) + \frac{159}{25} \sin(x) + \frac{113}{25} e^{-x} \cos(x) + \frac{9}{25} e^{-x} \sin(x) \]

> plot(y,x=0..10);

The assignment proper begins here. Use MAPLE as above to solve the problem,

\[ \frac{d^4 y}{dx^4} - \frac{d^3 y}{dx^3} - \frac{d^2 y}{dx^2} + \frac{dy}{dx} = x^2 + e^x \]

with \( y(0)=1, \ y'(0)=2, \ y''(0)=3, \) and \( y'''(0)=4. \) Plot the solution on an interval from \( x=-3 \) to \( x=2. \) Does the solution have a zero near \( x=-1? \)

Remarks: If you assign \( y:=1/5 \) in a worksheet, then MAPLE will replace \( y \) by \( 1/5 \) from that point on. You can unassign this value by typing \( y := 'y' ; \)

I assigned \( a_1, a_2, c_1 \) through \( c_4, y\_homo, y\_p, \) and \( y\_gen \) above. Rather than unassign all these, I could just select NEW from the file menue and start over.

The simplify command was useful above. You might also like the expand command.