1. If $f(x) = e^{3+\sqrt{x}}$, then $f^{-1}(x) =$

A. $(\ln x)^2 - 3$

B. $\frac{1}{2}\ln x - 3$

C. x^2/e^3

D. $\frac{1}{2}\ln x + 3$

[✓ E.] $(-3 + \ln x)^2$

$$y = e^{3+\sqrt{x}} = f(x)$$

$$\text{Ln}\, y = 3 + \sqrt{x}$$

$$-3 + \text{Ln}\, y = \sqrt{x}$$

$$\underline{(-3+\text{Ln}\, y)^2} = x = f^{-1}(y)$$

So $f^{-1}(x) = (-3 + \text{Ln}\, x)^2$

2. The domain of the function $f(x) = \ln(2 - \sqrt{x})$ is

A. $[0, \infty)$

B. $[4, \infty)$

C. $[0, \sqrt{2})$

[✓ D.] $[0, 4)$

E. $(-\infty, 4]$

↑ need $x \geq 0$ for $\sqrt{\ }$

need $2 - \sqrt{x} > 0$ in Ln

$$2 > \sqrt{x}$$

$$4 > x$$

So we need $0 \leq x < 4$, i.e.,

$$x \in [0, 4)$$

3. Let $f(x) = x^2 - x$. Find the difference quotient $\frac{f(2+h) - f(2)}{h}$. $=$

- **A.** $h + 1$
- **B.** $2h - 1$
- [✓ **C.**] $h + 3$
- **D.** $h^2 - h - 2$
- **E.** $h^2 + 1$

$$\frac{\left[(2+h)^2 - (2+h)\right] - \left[2^2 - 2\right]}{h}$$

$$= \frac{\cancel{4} + 4h + h^2 - \cancel{2} - h - \cancel{4} + \cancel{2}}{h}$$

$$= \frac{3h + h^2}{h} = 3 + h$$

4. Find a value for the constant c that makes $f(x)$ continuous for all values of x.

$$f(x) = \begin{cases} 6 & \text{if } x = 9 \\ \dfrac{x+c}{\sqrt{x} - 3} & \text{if } x \neq 9 \end{cases} \quad \leftarrow \text{continuous for } x \neq 9$$

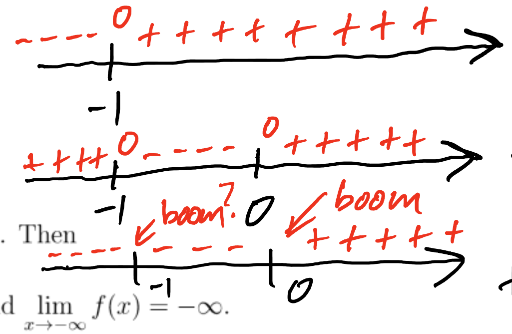
- A. $c = 3$
- [✓ B.] $c = -9$
- C. $c = 6$
- D. $c = -3$
- E. No value of c will make the function f continuous everywhere.

Need $x + c = 0$ at $x = 9$ to make limit at 9 exist, i.e., need $9 + c = 0$. So we need $c = -9$.

$$\frac{x-9}{\sqrt{x}-3} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} = \frac{(\cancel{x-9})(\sqrt{x}+3)}{\cancel{x-9}}$$

$$\to \sqrt{9} + 3 = 6 \text{ as } x \to 9. \ \checkmark$$

5. Suppose $f(x) = \frac{x^3+1}{x^2+x}$. Then



$f(x) = \frac{(\approx 1)}{(\text{small} < 0)}$
as $x \to 0-$

[✓ A.] $\lim_{x\to 0^-} f(x) = -\infty$ and $\lim_{x\to -\infty} f(x) = -\infty$.

B. $\lim_{x\to 0^-} f(x) = \infty$ and $\lim_{x\to -\infty} f(x) = \infty$.

C. $\lim_{x\to 0^-} f(x) = \infty$ and $\lim_{x\to -\infty} f(x) = -\infty$.

D. $\lim_{x\to 0^-} f(x) = -\infty$ and $\lim_{x\to -\infty} f(x) = \infty$.

E. None of the above.

$\lim_{x\to 0-} f(x) = -\infty$

boss

$$\frac{x^3+1}{x^2+x} = \frac{x^3\left(1+\frac{1}{x^3}\right)}{x^2\left(1+\frac{1}{x}\right)} = x\cdot\left[\begin{array}{c}\text{close}\\ \text{to } 1\end{array}\right] \text{ as } x\to-\infty.$$

boss

So $\lim_{x\to-\infty} f(x) = -\infty$

6. The limit

$$\lim_{h\to 0} \frac{\cos(h^2)-1}{h}$$

represents $f'(a)$, the derivative of some function f at some number a. Find such an f and a.

A. $f(x) = \cos(x^2)$, $a = 1$

B. $f(x) = \cos x$, $a = 0$

C. $f(x) = \cos x$, $a = 1$

[✓ D.] $f(x) = \cos(x^2)$, $a = 0$

E. None of the above.

$$\frac{\cos(h^2)-1}{h} = \frac{\cos(0+h)^2 - \cos 0^2}{h}$$

$$= \frac{f(a+h)-f(a)}{h}$$

Where $f(x) = \cos x^2$ and $a = 0$.

7. Solve $\ln(x+1) - \ln x = 1$ for x. $x =$

A. $\frac{e}{e-1}$

[✓ B.] $\frac{1}{e-1}$

C. $\frac{e-1}{e}$

D. $\frac{1-e}{e}$

E. $\frac{1}{1-e}$

$$\ln(x+1) - \ln x = \ln \frac{x+1}{x} = 1$$

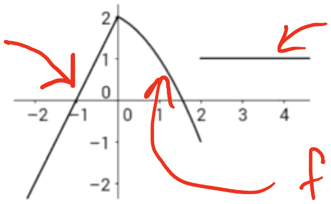
So $\frac{x+1}{x} = e^1 = e$

$$x+1 = xe$$

$$x(1-e) = -1$$

$$x = \frac{-1}{1-e} = \frac{1}{e-1}$$

8. Here is the graph of f:



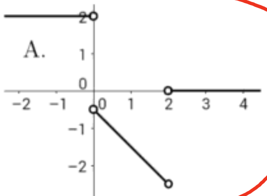
$f' = 2$

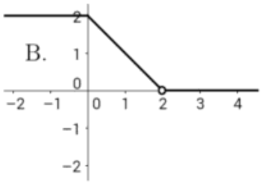
$f' = 0$

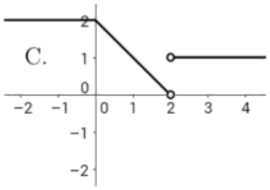
f' starts < 0 and gets more < 0

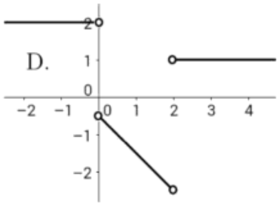
Find the graph of f', its derivative.

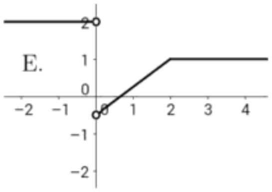
[✓ A.]











9. Compute the limit: $\lim_{x\to -1^+} \frac{x-4}{x^2(x+1)}$.

A. 0

B. -1

C. 1

D. ∞

[✓ E.] $-\infty$

$$\frac{x-4}{x^2(x+1)} = \underbrace{\frac{1}{x+1}}\cdot\underbrace{\left[\frac{x-4}{x^2}\right]}$$

$x+1$ positive and $\to 0$ as $x \to -1$ from the right

close to -5 when x close to -1

So $\lim_{x\to -1+} f(x) = -\infty$.

10. Compute the limit: $\lim_{x\to 3} \frac{x-3}{x^2-x-6}$. Which interval is the answer in?

A. $(-3,-2)$

B. $(-2,-1)$

C. $(-1,0)$

[✓ D.] $(0,1)$

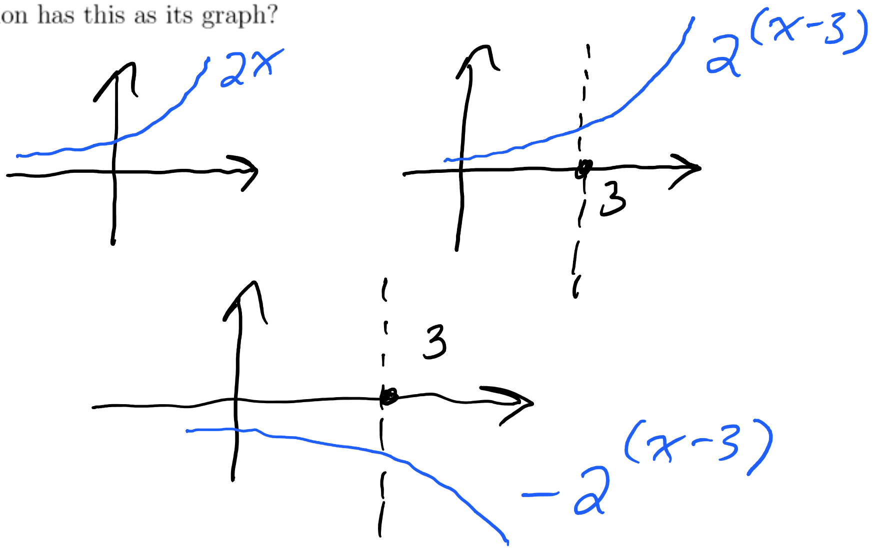
E. The limit does not exist.

$$\frac{\cancel{x-3}}{\cancel{(x-3)}(x+2)} = \frac{1}{x+2}$$

$$\xrightarrow[\text{as } x\to 3]{} \frac{1}{3+2} = \frac{1}{5} \in (0,1)$$

11. The graph of $y = 2^x$ is moved horizontally 3 units to the right and then reflected across the x axis. What function has this as its graph?

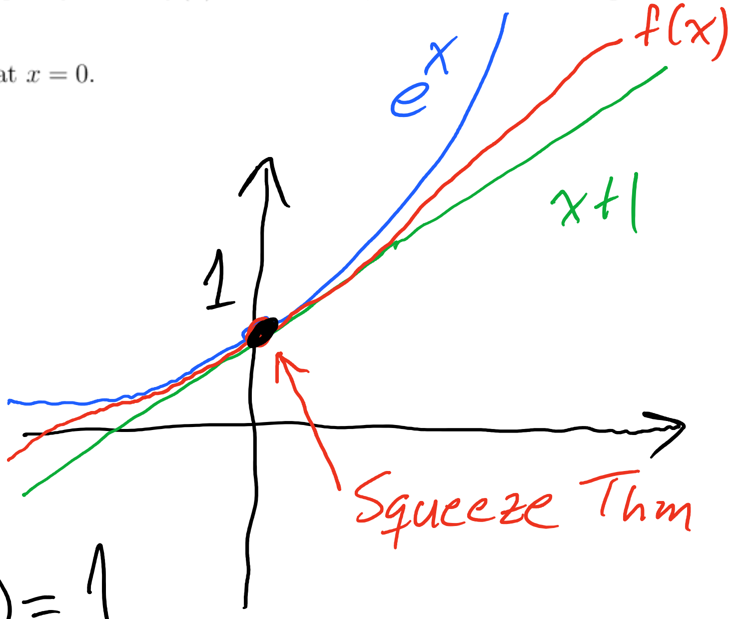
A. $y = 2^{x-3}$
B. $y = 2^{-x+3}$
[✓ C.] $y = -2^{x-3}$
D. $y = 2^{-x-3}$
E. $y = -2^{x+3}$



12. If $f(x)$ satisfies the inequality $x+1 \le f(x) \le e^x$ for all x, which of the following statements must be true?

✓ 1. $f(x)$ is continuous at $x = 0$.
✓ 2. $\lim_{x\to\infty} f(x) = \infty$.
3. $\lim_{x\to-\infty} f(x) = -\infty$. ✗
4. $\lim_{x\to-\infty} f(x) = 0$. ✗

[✓ A.] 1 and 2
B. 2 and 3
C. 2 and 4
D. 1 and 3
E. 1 and 4



$\lim_{x\to 0} f(x) = 1$
$= f(0)$ ← f continuous at 0.

$f(x) > x+1$ shows $\lim_{x\to\infty} f(x) = \infty$