1. If $y = (x^2 - 1)(2x+1)^2$, then $\frac{dy}{dx} =$ $2x(2x+1)^2 + (x^2-1)2(2x+1)\cdot 2$

A. $2(2x+1)(3x^2+x-1)$

B. $8x(2x+1)$

C. $2(2x+1)(x+2)$

D. $\boxed{2(2x+1)(4x^2+x-2)}$

E. $(2x+1)(10x^2-4)$

$$= 2(2x+1)\left[x(2x+1) + 2(x^2-1)\right]$$

$$= 2(2x+1)\left[2x^2 + x + 2x^2 - 2\right]$$

$$= 2(2x+1)\left[4x^2 + x - 2\right]$$

2. If $y = \sqrt{\sin 3x}$, then $y' =$ $\frac{1}{2}(\sin 3x)^{-1/2} \cdot (\cos 3x) \cdot 3$

A. $\dfrac{1}{2\sqrt{\sin 3x}}$

B. $3\sqrt{\cos 3x}$

C. $\boxed{\dfrac{3\cos 3x}{2\sqrt{\sin 3x}}}$

D. $\frac{3}{2}\sqrt{\sin 3x \cos 3x}$

E. $\dfrac{3}{2\sqrt{\cos 3x}}$

$$= \frac{3\cos 3x}{2\sqrt{\sin 3x}}$$

3. Find the slope of the tangent line to the curve

$$\sin(x+y) = xy$$

at the point $(0,0)$.

A. 0

B. 1

C. $\boxed{-1}$

D. $\frac{1}{2}$

E. It does not exist.

$$\frac{d}{dx}\sin(x+y) = \frac{d}{dx}(xy)$$

$$\cos(x+y)\cdot\left[1+\frac{dy}{dx}\right] = 1\cdot y + x\frac{dy}{dx}$$

$$\frac{dy}{dx}\left[\cos(x+y) - x\right] = y - \cos(x+y)$$

$$\frac{dy}{dx} = \frac{y-\cos(x+y)}{\cos(x+y)-x} = \frac{0-\cos 0}{(\cos 0)-0} = \frac{-1}{1} = -1$$

at $(0,0)$.

4. Suppose $g(e) = 4$ and $g'(e) = 2$. If $y = x^{g(x)}$, then what is y' at $x = e$?

A. $\frac{4}{e} + 2e^4$

B. $4e^3$

C. $8e^3$

D. $\boxed{2e^4 + 4e^3}$

E. $\frac{4}{e}$

$$\ln y = \ln x^{g(x)} = g(x)\ln x$$

$$\frac{1}{y}\frac{dy}{dx} = g'(x)\ln x + g(x)\cdot\frac{1}{x}$$

$$\frac{dy}{dx} = x^{g(x)}\left[g'(x)\ln x + g(x)\cdot\frac{1}{x}\right]$$

At $x=e$: $$\frac{dy}{dx} = e^{g(e)}\left[g'(e)\underbrace{\ln e}_{1} + g(e)\cdot\frac{1}{e}\right]$$

$$= e^4\left[2\cdot 1 + 4\cdot\frac{1}{e}\right] = 2e^4 + 4e^3$$

5. A certain bacteria culture grows at a rate proportional to its size and has a doubling time of two hours. How long does it take for the population to triple (i.e. grow to three times its initial size)?

A. 3 hours

B. $2\ln\left(\frac{3}{2}\right)$ hours

C. $\frac{3\ln 2}{\ln 3}$ hours

D. $\frac{3}{2}\ln 2$ hours

E. $\boxed{\frac{2\ln 3}{\ln 2} \text{ hours}}$

$P = P_0 e^{kt}$ $D =$ doubling time $= 2$

$2P_0 = P_0 e^{kD}$ $2 = e^{kD}$ $\ln 2 = kD = k\cdot 2$

so $k = \frac{\ln 2}{2}$

$3P_0 = P_0 e^{\frac{\ln 2}{2}\cdot t}$

$\ln 3 = \frac{\ln 2}{2}\cdot t$, P triples when

$t = \frac{2\ln 3}{\ln 2}$

6. If $f(x) = \frac{x}{x+1}$, then $f''(1) =$

A. 0

B. 1

C. -1

D. $\frac{1}{4}$

E. $\boxed{-\frac{1}{4}}$

$f'(x) = \frac{1\cdot(x+1) - 1\cdot x}{(x+1)^2} = \frac{1}{(x+1)^2}$

$f''(x) = -2(x+1)^{-3}\cdot 1 = \frac{-2}{(x+1)^3}$

$f''(1) = \frac{-2}{2^3} = \frac{-1}{2^2} = \frac{-1}{4}$

$(\ln 1 = 0)$

7. If $f(x) = x^2 e^{2x} \ln x$, then $f'(1) =$

A. $\boxed{e^2}$

B. $2e^2$

C. $4e^2$

D. $2 + 2e^2$

E. $2 + 2e^2 + e$

$$f'(x) = 2x e^{2x} \ln x + x^2 2e^{2x} \ln x + x^2 e^{2x} \cdot \frac{1}{x}$$

$$f'(1) = 2 \cdot 1 \cdot e^2 \cdot 0 + 1^2 2e^2 \cdot 0 + 1 \cdot e^2 \cdot \frac{1}{1}$$

$$= 0 + 0 + e^2$$

8. If $0 < x < \frac{1}{2}$, then $\sec(\underbrace{\sin^{-1} 2x}_{\theta}) =$

A. $\boxed{\dfrac{1}{\sqrt{1-4x^2}}}$

B. $\dfrac{1}{\sqrt{1+4x^2}}$

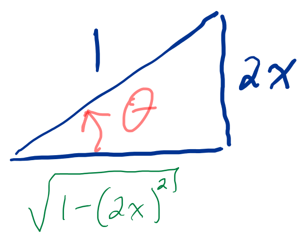
C. $\sqrt{1+4x^2}$

D. $\dfrac{2x}{\sqrt{1-4x^2}}$

E. $\dfrac{\sqrt{1-4x^2}}{2x}$

$$\sin\theta = \frac{2x}{1} = 2x$$

$$\text{so } \theta = \sin^{-1} 2x$$



$$\sec\theta = \frac{1}{\sqrt{1-4x^2}}$$

$$\underline{\text{or}} \quad \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\sqrt{1-\sin^2\theta}} = \frac{1}{\sqrt{1-(2x)^2}}$$

9. Evaluate $\displaystyle\lim_{x\to 0} \frac{\sin 2x}{x(2+\sin x)^4}$.

A. $\frac{1}{16}$

B. $\boxed{\frac{1}{8}}$

C. $\frac{1}{2}$

D. 2

E. ∞

$$2\,\frac{\sin 2x}{2x} \cdot \frac{1}{(2+\sin x)^4}$$

as $x \to 0$

$$2 \cdot 1 \cdot \frac{1}{(2+\underbrace{\sin 0}_{0})^4}$$

$$= \frac{2}{2^4} = \frac{1}{2^3} = \frac{1}{8}$$

10. If $y = (\cos x)^4$, what is $\frac{dy}{dx}$ when $x = \frac{4\pi}{3}$?

A. $\frac{-3\sqrt{3}}{4}$

B. $\boxed{\frac{-\sqrt{3}}{4}}$

C. $\frac{1}{4}$

D. $\frac{\sqrt{3}}{4}$

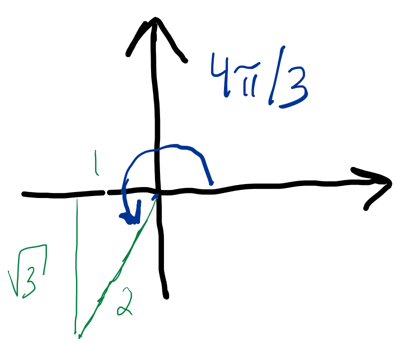
E. $\frac{3\sqrt{3}}{4}$

$$\frac{dy}{dx} = 4(\cos x)^3(-\sin x)$$

at $x = \frac{4\pi}{3}$:

$$\frac{dy}{dx} = 4\left(-\frac{1}{2}\right)^3\left(-\left(-\frac{\sqrt{3}}{2}\right)\right)$$

$$= -\frac{4}{2^3}\cdot\frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{4}$$



11. A ball is thrown vertically upward at a velocity of 10 ft/sec from a point 5 ft above the surface of an alien planet. Its height (in feet) after t seconds is

$$s(t) = 5 + 10t - 40t^2.$$

Which of the following statements are true?

I. The ball is slowing down when $0 < t < 1/2$.

II. The ball returns to the surface with a speed of 30 ft/sec.

III. The acceleration is a constant -80 ft/sec^2.

A. Only one of the statements is true.

B. I and II

C. I and III

D. $\boxed{\text{II and III}}$

E. All three statements are true.

$s'(t) = 10 - 80t$

$s''(t) = -80$ III true.

Ball hits: $5 + 10t - 40t^2 = 0$

$1 + 2t - 8t^2 = 0$

$(1+4t)(1-2t) = 0$

$t = -\frac{1}{4}, \frac{1}{2}$

$s'(\frac{1}{2}) = 10 - 80 \cdot \frac{1}{2} = -30$

So speed $= 30$. II true

Ball peaks when $s'(t) = 0$

$10 - 80t = 0$

$t = 1/8$

Slows down $0 < t < 1/8$,

but speeds up $1/8 < t < \frac{1}{2}$. I false.

12. The line tangent to the curve $y = \dfrac{1}{x^2}$ at $(1, 1)$ crosses the x-axis at $x =$

A. $\frac{1}{2}$

B. $\boxed{\frac{3}{2}}$

C. 2

D. $\frac{5}{2}$

E. 4

$\frac{dy}{dx} = -2x^{-3} = -2 \cdot 1^{-3} = -2$ when $x = 1$.

Tangent line $\frac{y-1}{x-1} = -2$, $y = 1 - 2(x-1)$

Crosses when $1 - 2(x-1) = 0$

$(x-1) = \frac{1}{2}$

$x = \frac{3}{2}$