1. If \( f(1) = 3 \) and \( f'(1) = 5 \), use a linear approximation to estimate \( f(0.99) \).

\[
L(x) = f(a) + f'(a)(x-a) \\
a = 1 \\
x = 0.99
\]

\[
L(0.99) = 3 + 5 (0.99-1) = 3 - 0.05 = 2.95
\]

2. The graph of the first derivative \( f'(x) \) of a function \( f(x) \) is shown. On what intervals is \( f(x) \) decreasing?

A. \((-1, 1) \cup (2, 3) \cup (5, \infty)\)
B. \((-\infty, -1) \cup (1, 2) \cup (3, 5)\)
C. \((-\infty, 0) \cup (1.5, 2.5) \cup (4.5, 5.5)\)
D. \((-2, 0) \cup (4, \infty)\)
E. \((-\infty, -2) \cup (0, 2) \cup (2, 4)\)
3. Let \( f(x) = -x^3 + 3x + 6 \). Let \( M \) be the absolute maximum value of \( f(x) \) and \( m \) the absolute minimum value of \( f(x) \) on \([-2, 3]\). What is \( M - m \)?

A. 4  
B. 28  
C. [20]  
D. 8  
E. 16

\[
f'(x) = -3x^2 + 3 = -3(x^2 - 1) = 0 \quad \text{when} \quad x = \pm 1
\]

Critical numbers in \([-2, 3]\): \( c = \pm 1 \)

\[
\begin{array}{c|cc}
 x & f(x) & \text{M} \\
-2 & 8 - 6 + 6 = 8 & \leftarrow M \\
-1 & 1 - 3 + 6 = 4 & \leftarrow M \\
1 & -1 + 3 + 6 = 8 & \leftarrow M \\
3 & -27 + 9 + 6 = -12 & \leftarrow m
\end{array}
\]

\( M - m = 8 - (-12) = 20 \)

4. What is the maximum value of \( f(x) = \sqrt{3}\sin x + \cos x \) on \([0, \pi]\)?

A. \( \sqrt{3} \)  
B. \( 1/2 \)  
C. 1  
D. \( [2] \)  
E. \( \sqrt{3} + 1 \)

\[
f(x) = \sqrt{3}\cos x - \sin x = 0 \quad \text{when} \quad \frac{\sin x}{\cos x} = \sqrt{3}
\]

Only one critical number in \([0, \pi]\): \( c = \frac{\pi}{3} \)

\[
\begin{array}{c|c}
 x & f(x) & \text{Max} \\
0 & \sqrt{3}\cdot0 + 1 = 1 & \leftarrow \text{Max} \\
\frac{\pi}{3} & \sqrt{3}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{3}{2} & \leftarrow \text{Max} \\
\pi & \sqrt{3}\cdot0 - 1 = -1 \\
\end{array}
\]
5. A particle moves along the x axis with a position function $x(t)$ with $x(0) = 3$. What is the largest possible positive value for $x(5)$ if $x'(t) \leq 7$ for all $t \geq 0$?

A. 7  
B. 10  
C. 32  
D. [38]  
E. 5

MVT: $\frac{x(5) - x(0)}{5-0} = \frac{x'(c)}{c}$ for some $c$ in $(0,5)$

So $\frac{x(5) - 3}{5} \leq 7$

$x(5) - 3 \leq 35$

$x(5) \leq 38$  \(\text{happens if } x(t) = 3 + 7t\)

6. If $y$ is a function of $x$ such that $y' > 0$ for all $x$ and $y'' < 0$ for all $x$, which of the following could be part of the graph of $y = f(x)$?

- [Images of graphs with annotations: increasing, decreasing, not C.D., not increasing, concave down]
7. The graph of a twice-differentiable function \( f \) is show in the figure below. Which of the following is true?

\[ f(1) < f'(1) < f''(1) \]

A. \( f(1) < f'(1) < f''(1) \)
B. \( f(1) < f''(1) < f'(1) \)
C. \( f'(1) < f(1) < f''(1) \)
D. \( f''(1) < f(1) < f'(1) \)
E. \( f''(1) < f'(1) < f(1) \)

8. If \( f(3) = 5, f'(3) = 0, \) and \( f''(3) = -2, \) which of the following statements must be true about the point \( (3,5) \)? You may assume that \( f(x), f'(x), \) and \( f''(x) \) are continuous for all \( x \).

I. \( f \) has a local maximum there.
II. \( f \) has a local minimum there.
III. The graph is concave upward in the neighborhood of \((3,5)\).
IV. The graph is concave downward in the neighborhood of \((3,5)\).
V. The tangent line at \((3,5)\) is horizontal.

A. II, III, and V
B. I, IV, and V
C. II, IV, and V
D. I and III
E. II and IV

\[ f''(1) < 0. \]
\[ \text{x-intercept: } f(1) = 0 \]
\[ f \text{ increasing at } 1: f'(1) > 0 \]

\[ f''(3) = -2: \text{ Concave down} \]

Second Der. Test: \((3,5)\) a local max.
9. Consider the function \( f(x) = xe^{2x} \). Which of the following statements are true?

- \( f'(x) = e^{2x} + x(2e^{2x}) \)  
- \( f''(x) = 2e^{2x} + 4xe^{2x} = 4(1+x)e^{2x} \)

A. (1), (2), and (3)  
B. (3) only  
C. (1) and (2)  
D. (1) only  
E. (2) and (3)

---

10. Compute the limit \( \lim_{x \to 0} (1+2x)^{3/x} \).

A. 3  
B. 6  
C. \( 3e^2 \)  
D. \( e^3 \)  
E. \( e^6 \)

\[
\ln y = \frac{3}{x} \ln (1+2x)  \\
\lim_{x \to 0} \ln y = \lim_{x \to 0} \frac{3 \ln (1+2x)}{x} = \frac{0}{0} 
\]

\[
= \lim_{x \to 0} \frac{3 \cdot (1+2x) \cdot 2}{1} = 6 
\]

\( y = e^{\ln y} \to e^6 \) as \( x \to 0 \).
11. Compute the limit \( \lim_{x \to 0^+} \frac{\sin x}{e^x} = \frac{\sin 0}{e^0} = \frac{0}{1} = 0 \)

- A. 0
- B. 1
- C. \( \infty \)
- D. \( \frac{1}{e} \)
- E. DNE

L'Hopital's rule is applied.

12. Find \( \lim_{x \to 0^+} x(\ln x)^2 \).

\[
\lim_{x \to 0^+} \frac{(\ln x)^2}{\frac{1}{x}} = \lim_{x \to 0^+} \frac{2(\ln x) \cdot \frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x \to 0^+} \frac{2\ln x}{\frac{-1}{x}} = \lim_{x \to 0^+} \frac{2 \cdot \frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x \to 0^+} 2x = 0
\]

- A. 1
- B. -2
- C. 2
- D. \( \infty \)
- E. 0