$$\frac{\sqrt{t^2+16}-4}{t^2}\cdot\frac{\sqrt{t^2+16}+4}{\sqrt{t^2+16}+4}=$$

1. Find the limit.

$$\lim_{t\to 0}\frac{\sqrt{t^2+16}-4}{t^2}$$

$$\frac{(t^2+16)-16}{t^2\left(\sqrt{t^2+16}+4\right)}$$

A. 0
[✓ B.] $\frac{1}{8}$
C. $\frac{1}{4}$
D. ∞
E. -4

$$=\frac{t^2}{t^2\left(\sqrt{t^2+16}+4\right)}=$$

$$\frac{1}{\sqrt{t^2+16}+4}\underset{\substack{\text{as}\\ x\to 0}}{\longrightarrow}\frac{1}{\sqrt{0^2+16}+4}=\frac{1}{8}$$

2. Where is the function $\frac{\ln x}{x^2-1}$ continuous?

need $x>0$

need $x\neq\pm1$

[✓ A.] $(0,1)\cup(1,\infty)$
B. $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$
C. $(0,\infty)$
D. $(-\infty,-1)\cup(-1,0)\cup(0,1)\cup(1,\infty)$
E. $(-1,1)$

Note: $\lim_{x\to 1}\frac{\ln x}{x^2-1}\underset{\left(\frac{0}{0}\right)}{\overset{L'H}{=}}\lim_{x\to 1}\frac{\left(\frac{1}{x}\right)}{2x}=\frac{1}{2}$,

but the function is not defined at $x=1$.

3. If an item is thrown upward on Mars, with velocity 10 m/s, its height in meters after t seconds is given by $y = 10t - 2t^2$. Find the average velocity over the time interval $[1, 3]$.

A. 1 m/s
B. 20 m/s
[✓ C.] 2 m/s
D. 0 m/s
E. 10 m/s

$$\text{Average velocity} = \frac{y(3)-y(1)}{3-1}$$

$$= \frac{(10\cdot 3 - 2\cdot 3^2) - (10\cdot 1 - 2\cdot 1^2)}{3-1}$$

$$= \frac{(30-18)-(10-2)}{2}$$

$$= \frac{12-8}{2} = \frac{4}{2} = 2 \text{ m/sec}$$

4. Find the approximate value of $\ln(0.95)$ by considering a linearization.

A. 0.95
[✓ B.] -0.05
C. $-\frac{1}{19}$
D. -0.95
E. $\frac{20}{19}$

$f(x) = \text{Ln } x$ $\qquad$ $\underline{\text{Take } a=1}$

$f'(x) = \frac{1}{x}$

$$f(x) \approx f(a) + f'(a)(x-a)$$

$$\text{Ln}(.95) \approx (\text{Ln } 1) + \left(\tfrac{1}{1}\right)(.95-1)$$

$$= 0 + 1\cdot(-0.05) = -0.05$$

$= e^u$ where $u = x^2$.

5. Suppose $f(x) = e^{x^2}$ Find $f''(x)$.

$$f'(x) = e^u \cdot \frac{du}{dx} = e^{x^2}(2x)$$

- A. $4x^2e^{x^2}$
- B. $8x^2e^{x^2}$
- C. $2xe^{2x}$
- D. $(2+2x)e^{x^2}$
- [✓ E.] $(2+4x^2)e^{x^2}$

$$f''(x) = \left[2xe^{x^2}\right](2x) + e^{x^2}\cdot(2)$$

$$= 4x^2e^{x^2} + 2e^{x^2}$$

6. Find the slope of the tangent line to $f(x) = \dfrac{1}{1+\sqrt{x}}$ at $x = 1$.

$= \dfrac{1}{u} = u^{-1}$ where $u = 1 + x^{1/2}$

- [✓ A.] $-\frac{1}{8}$
- B. 2
- C. $-\frac{1}{4}$
- D. $\frac{1}{2}$
- E. 1

$$f'(x) = -u^{-2}\cdot\frac{du}{dx} = \frac{-1}{u^2}\cdot\left(0+\frac{1}{2}x^{-1/2}\right)$$

$$= \frac{-1}{(1+\sqrt{x})^2}\cdot\left(\frac{1}{2\sqrt{x}}\right)$$

Slope $= f'(1) = \dfrac{-1}{(1+\sqrt{1})^2}\cdot\left(\dfrac{1}{2\sqrt{1}}\right) = \dfrac{-1}{2^3} = \dfrac{-1}{8}$

7. f is a function with $f(1) = 2$, $f'(1) = 3$, $f\left(\frac{\pi}{4}\right) = 1$, and $f'\left(\frac{\pi}{4}\right) = 4$. If

$$h(x) = f(\tan x),$$

find $\frac{dh}{dx}$ at $x = \frac{\pi}{4}$.

[✓ A.] 6

B. 8

C. 3

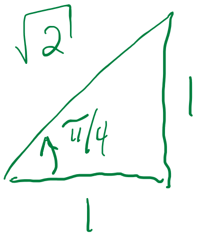
D. 4

E. 2

$$h(x) = f(u) \text{ where } u = \operatorname{Tan} x$$

$$h'(x) = f'(u) \cdot \frac{du}{dx}$$

$$= f'(u) \operatorname{Sec}^2 x$$



$$h'(x) = f'(\operatorname{Tan} x) \operatorname{Sec}^2 x$$

$$h'\left(\frac{\pi}{4}\right) = f'\left(\operatorname{Tan}\frac{\pi}{4}\right)\operatorname{Sec}^2\frac{\pi}{4} = \underbrace{f'(1)}_{3}\left(\sqrt{2}\right)^2 = 3 \cdot 2$$

$$= 6$$

8. Find the equation of the tangent line to the curve $2y^4 - x^2y = x^3$ at the point $(1, 1)$.

A. $y = \frac{1}{3}x + \frac{2}{3}$

[✓ B.] $y = \frac{5}{7}x + \frac{2}{7}$

C. $y = \frac{5}{8}x + \frac{3}{8}$

D. $y = x$

E. $y = \frac{5}{9}x + \frac{4}{9}$

$$2 \cdot 4y^3 \cdot \frac{dy}{dx} - \left(2xy + x^2\frac{dy}{dx}\right) = 3x^2$$

$$\frac{dy}{dx}\left(8y^3 - x^2\right) = 3x^2 + 2xy$$

$$\frac{dy}{dx} = \frac{3x^2 + 2xy}{8y^3 - x^2} = \frac{3 \cdot 1^2 + 2 \cdot 1 \cdot 1}{8 \cdot 1^3 - 1^2}$$

$$= \frac{3+2}{8-1} = \frac{5}{7} \quad \text{when } x = 1, y = 1.$$

↑ slope tangent line

Tangent line

$$\frac{y-1}{x-1} = \frac{5}{7}$$

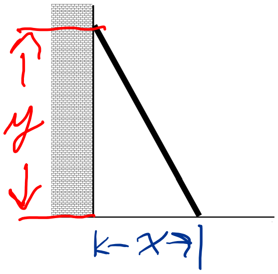
$$y = 1 + \frac{5}{7}(x-1)$$

$$= \frac{5}{7}x + \frac{2}{7}$$

9. A ladder of length 10 m is resting against a vertical wall. If the foot of the ladder slides away from the wall at a rate of 3 m/s, how fast is the top of the ladder sliding down the wall at the instant the foot of the ladder is 8m from the wall?

Given: $x^2+y^2=10^2$ | Want $\frac{dy}{dt}$

$\frac{dx}{dt}=3$ | when $x=8$.



A. $\frac{25}{3}$ m/s

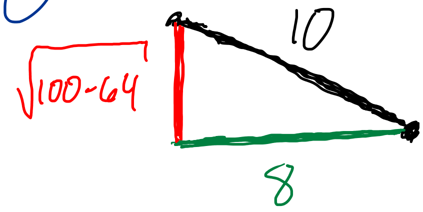
B. 3 m/s

[✓ C.] 4 m/s

D. $\frac{8}{3}$ m/s

E. $\frac{15}{2}$ m/s

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$



$$x\frac{dx}{dt}+y\frac{dy}{dt}=0$$

$$(8)\cdot 3+\sqrt{36}\cdot\frac{dy}{dt}=0 \quad \text{when } \begin{cases} x=8 \\ y=\sqrt{36} \end{cases}$$

$\frac{dy}{dt}=\frac{-24}{\sqrt{36}}=-4$ m/s. Rate sliding <u>down</u> is +4 m/s.

10. Where is the function $f(x)=x^4-2x^3+4x$ concave down?

A. $(-\infty, 0)$

B. $(-\sqrt{2}, \sqrt{2})$

C. $(1, \infty)$

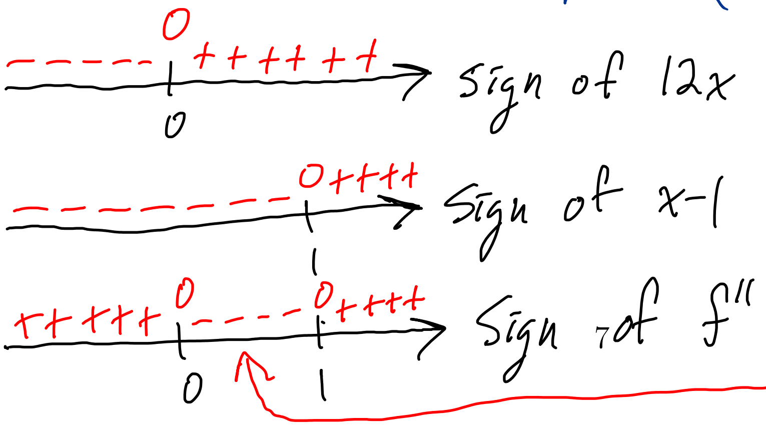
[✓ D.] $(0, 1)$

E. $(-\infty, \sqrt{2})$

$$f'(x)=4x^3-6x^2+4$$

$$f''(x)=12x^2-12x$$

$$=12x(x-1)$$



Concave down on $(0,1)$

11. Find the horizontal asymptotes of

$$f(x) = \frac{e^x + 2e^{-x}}{e^x + 3e^{-x}}.$$

A. $y = -1$ and $y = 1$

B. $y = -\frac{2}{3}$ and $y = \frac{2}{3}$

C. $y = 1$

[✓ D.] $y = \frac{2}{3}$ and $y = 1$

E. There are no horizontal asymptotes.

e^x boss when $x \to \infty$:

$$\frac{e^x + 2e^{-x}}{e^x + 3e^{-x}} = \frac{\cancel{e^x}}{\cancel{e^x}} \frac{[1 + 2e^{-2x}]}{[1 + 3e^{-2x}]} \longrightarrow \frac{1+0}{1+0} = 1 \text{ as } x \to \infty$$

$y = 1$ horiz asymp

e^{-x} boss as $x \to -\infty$:

$$\frac{e^x + 2e^{-x}}{e^x + 3e^{-x}} = \frac{\cancel{e^{-x}}}{\cancel{e^{-x}}} \frac{[e^{2x} + 2]}{[e^{2x} + 3]} \longrightarrow \frac{0+2}{0+3} = \frac{2}{3} \text{ as } x \to -\infty$$

$y = \frac{2}{3}$ horiz asymp

12. Find the limit.

$$\lim_{x \to 0} \frac{\csc(2x)}{\cot x}$$

[✓ A.] $\frac{1}{2}$

B. 1

C. 2

D. ∞

E. The limit does not exist.

$$\frac{\csc 2x}{\cot x} = \frac{\left(\frac{1}{\sin 2x}\right)}{\left(\frac{\cos x}{\sin x}\right)}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\sin 2x} = \frac{1}{\cos x} \cdot \frac{\sin x}{x} \cdot \frac{2x}{\sin 2x} \cdot \frac{1}{2}$$

$$\longrightarrow \frac{1}{\cos 0} \cdot 1 \cdot 1 \cdot \frac{1}{2} = \frac{1}{2}$$

or use L'Hôpital's rule.

13. Find the vertical asymptotes of the function $f(x) = \frac{x^2+1}{3x-2x^2}$.

numerator always >0

A. $x = 2$ and $x = -3$

B. $x = -\frac{1}{2}$

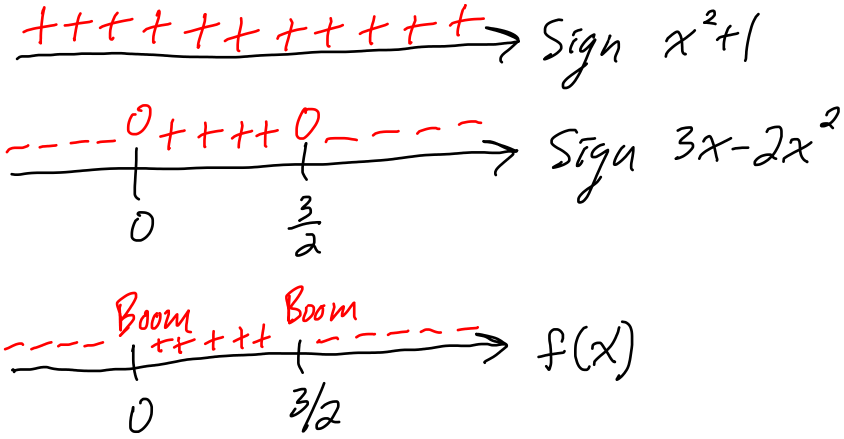
C. $x = \frac{3}{2}$

[✓ D.] $x = 0$ and $x = \frac{3}{2}$

E. $x = \frac{1}{3}$

$3x-2x^2 = x(3-2x)$

denom $=0$ at $x=0$, $x=\frac{3}{2}$.



$\lim_{x\to 0-} f(x) = -\infty$

$\lim_{x\to 0+} f(x) = \infty$

$\lim_{x\to 3/2-} f(x) = \infty$

$\lim_{x\to \frac{3}{2}+} f(x) = -\infty$

14. Let f be a function whose derivative, f', is given by

$$f'(x) = x(x-2)(x-1)^2.$$

The function f has

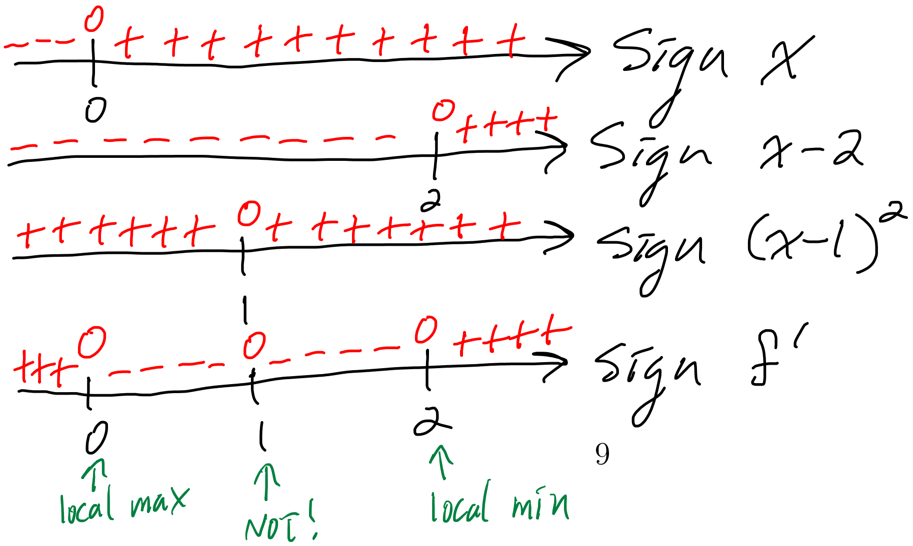
A. local maxima at $x = 0$ and $x = 2$.

[✓ B.] a local minimum at $x = 2$, and a local maximum at $x = 0$.

C. local minima at $x = 0$ and $x = 1$, and a local maximum at $x = 2$.

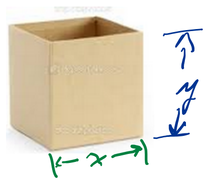
D. local minima at $x = 0$ and $x = 2$, and a local maximum at $x = 1$.

E. a local minimum at $x = 1$, and local maxima at $x = 0$ and $x = 2$.



15. If 1200 m^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

Given: $1200 = (x^2) + 4xy \leftarrow y = \dfrac{1200 - x^2}{4x}$



$$V = x^2 \cdot y = x^2\left(\frac{1200 - x^2}{4x}\right)$$

A. $2000\sqrt{2}$ m^3

B. 2000 m^3

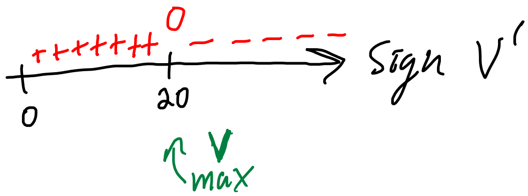
C. $2000\sqrt{3}$ m^3

D. $1000\sqrt{2}$ m^3

[✓ E. 4000 m^3]

$$V = x\left(300 - \frac{1}{4}x^2\right) = 300x - \frac{x^3}{4}$$

$$V' = 300 - \frac{3x^2}{4} = 0$$ when $x^2 = 400$: $\boxed{x = 20}$



When $x = 20$, $V = 300 \cdot 20 - \dfrac{(20)^3}{4}$

$= 6 \cdot 10^3 - \dfrac{8}{4} \cdot 10^3$

$= (6-2) \cdot 10^3 = 4000$ m^3

16. Find the point on the curve $y = \sqrt{x}$ which is closest to $(3, 0)$.

Hint: The distance between two points (a, b) *and* (x, y) *is* $\sqrt{(x-a)^2 + (y-b)^2}$.

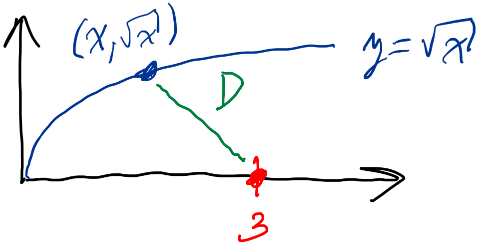
[✓ A. $\left(\frac{5}{2}, \sqrt{\frac{5}{2}}\right)$]

B. $(1, 1)$

C. $(3, \sqrt{3})$

D. $\left(\frac{3}{2}, \sqrt{\frac{3}{2}}\right)$

E. None of the above.



$$D = \sqrt{(x-3)^2 + (\sqrt{x} - 0)^2}$$

$$= \sqrt{(x-3)^2 + x}$$

$$\frac{dD}{dx} = \frac{1}{2}\left((x-3)^2 + x\right)^{-1/2}\left[2(x-3) + 1\right] = 0$$

when $x - 3 = -\frac{1}{2}$

$x = \frac{5}{2}$

Closest point $\left(\frac{5}{2}, \sqrt{\frac{5}{2}}\right)$

17. Find an antiderivative of the function $f(x) = \frac{2}{1+x^2}$.

- [✓ A.] $2\tan^{-1} x$
- B. $\ln(1+x^2)$
- C. $2x - \frac{2}{x}$
- D. $\frac{-2}{1+x}$
- E. $\frac{-4x}{(1+x^2)^2}$

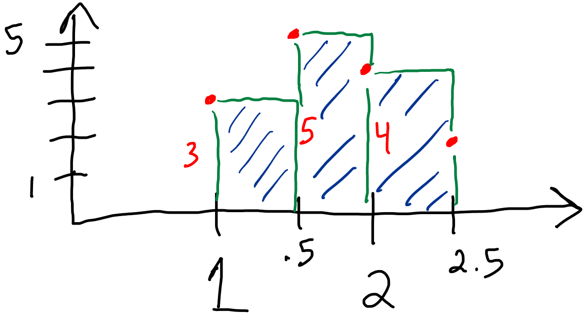
$$F(x) = \int \frac{2}{1+x^2}\,dx = 2\int \frac{1}{1+x^2}\,dx$$

$$= \underline{2\,\text{Tan}^{-1}x} + C = \text{all antideriv's}$$

18. Using the table of values below, estimate $\int_1^{2.5} g(x)\,dx$ using a *left* Riemann sum with $n = 3$ rectangles.

x	1	1.5	2	2.5
$g(x)$	3	5	4	2

- A. 12
- B. 14
- C. $\frac{28}{3}$
- D. $\frac{37}{2}$
- [✓ E.] 6



$$3(.5) + 5(.5) + 4(.5) = 12(.5) = 6$$

19. Suppose

$$f(x)=\int_0^{x^3}(t^2+1)\,dt.$$

Find $f'(x)$.

$$f(x)=\int_0^{u}(t^2+1)dt \text{ where } u=x^3.$$

A. $3x^4+3x^2$
[✓ B.] $3x^8+3x^2$
C. x^6+1
D. $6x^5$
E. $(x^2+1)^3$

$$f'(x)=f'(u)\cdot\frac{du}{dx}$$

$$=[u^2+1]\cdot 3x^2=[(x^3)^2+1]\cdot 3x^2$$

↑ Fund Thm Calc.

$$=(x^6+1)\cdot 3x^2$$

$$=3x^8+3x^2$$

20. Evaluate.

$$\int_1^4\frac{\sqrt{x}+x}{x^2}\,dx$$

A. $1+\ln 16$
B. $-3+\ln 4$
C. $\dfrac{31}{32}+\ln 16$
D. $\dfrac{3}{2}+\ln 16$
[✓ E.] $1+\ln 4$

$$=\int_1^4\frac{x^{1/2}}{x^2}+\frac{x}{x^2}\,dx=\int_1^4 x^{-3/2}+\frac{1}{x}\,dx$$

$$=\left[\frac{1}{-\frac{3}{2}+1}x^{-\frac{3}{2}+1}+\ln|x|\right]_1^4$$

$$=\left[-2x^{-1/2}+\ln x\right]_1^4=\left(\frac{-2}{\sqrt{4}}+\ln 4\right)-\left(\frac{-2}{\sqrt{1}}-\ln 1\right)$$

↑ $\ln 1=0$

$$=(-1+\ln 4)-(-2)$$

$$=1+\ln 4$$

$u=1-2x^4 \qquad du=-8x^3\,dx \qquad x^3dx=-\frac{1}{8}du$

21. $\int x^3(1-2x^4)^{1/4}\,dx$ $= \int \underbrace{(1-2x^4)^{1/4}}_{u^{1/4}} \underbrace{(x^3dx)}_{-\frac{1}{8}du}$

A. $-\frac{5}{32}(1-2x^4)^{5/4}+C$
B. $-\frac{1}{8}(1-2x^4)^{5/4}+C$
C. $\frac{4}{5}(1-2x^4)^{5/4}+C$
[✓ D.] $-\frac{1}{10}(1-2x^4)^{5/4}+C$
E. $\frac{2}{5}(1-2x^4)^{5/4}+C$

$$= -\frac{1}{8}\int u^{1/4}\,du = -\frac{1}{8}\left[\frac{1}{\frac{1}{4}+1}u^{\frac{1}{4}+1}\right]+C$$

$$= -\frac{1}{8}\left(\frac{4}{5}u^{5/4}\right)+C$$

$$= -\frac{1}{10}(1-2x^4)^{5/4}+C$$

22. $\int_{\ln\sqrt{3}}^{\ln\sqrt{8}} 2e^{2t}\sqrt{1+e^{2t}}\,dt$ $\qquad u=1+e^{2t} \qquad du=2e^{2t}\,dt$

[✓ A.] $\frac{38}{3}$
B. $\frac{32}{3}\sqrt{2}-2\sqrt{3}$
C. $\frac{1}{12}$
D. $2\sqrt{2}-\sqrt{3}$
E. $\frac{8\ln(3)-8}{5}$

When $x=\ln\sqrt{3}$, $u=1+e^{2\ln\sqrt{3}}=1+e^{\ln(\sqrt{3})^2}=1+3=4$

When $x=\ln\sqrt{8}$, $u=1+e^{2\ln\sqrt{8}}=1+e^{\ln(\sqrt{8})^2}=1+8=9$

$$\int_{x=\ln\sqrt{3}}^{\ln\sqrt{8}} \underbrace{(1+e^{2t})^{1/2}}_{u^{1/2}} \underbrace{(2e^{2t}dt)}_{du}$$

$$= \int_{u=4}^{9} u^{1/2}\,du = \left[\frac{1}{\frac{1}{2}+1}u^{1/2+1}\right]_4^9 = \frac{2}{3}u^{3/2}\Big|_1^9 = \frac{2}{3}\left(9^{3/2}-4^{3/2}\right)$$

$$= \frac{2}{3}(27-8) = \frac{38}{3}$$

23. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{3\pi}{4}} (\cos t + \sin t)\, dt$ $= \left[\sin t - \cos t\right]_{\pi/6}^{3\pi/4}$

A. $-\frac{\sqrt{3}}{2} - \frac{1}{2}$

B. $\sqrt{2} - \frac{\sqrt{3}}{2} + \frac{1}{2}$

[✓ C.] $\sqrt{2} + \frac{\sqrt{3}}{2} - \frac{1}{2}$

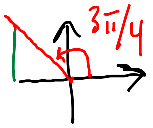
D. $\frac{\sqrt{3}}{2} + \frac{1}{2}$

E. $-\frac{\sqrt{3}}{2} + \frac{1}{2}$

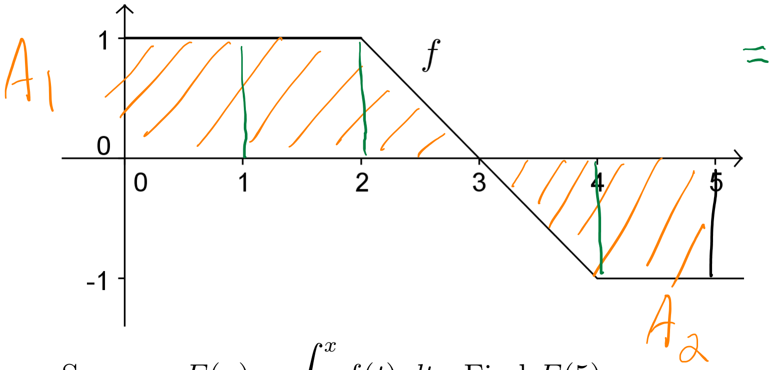
$= \left[\sin\frac{3\pi}{4} - \cos\frac{3\pi}{4}\right] - \left[\sin\frac{\pi}{6} - \cos\frac{\pi}{6}\right]$

$= \left[\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right] - \left[\frac{1}{2} - \frac{\sqrt{3}}{2}\right]$

$= \sqrt{2} - \frac{1}{2} + \frac{\sqrt{3}}{2}$



24. The graph of f is pictured below.



= 2 squares − 1 square

(Δ's cancel)

Suppose $F(x) = \displaystyle\int_0^x f(t)\, dt$. Find $F(5)$.

A. -1

B. 0

[✓ C.] 1

D. 4

E. -2

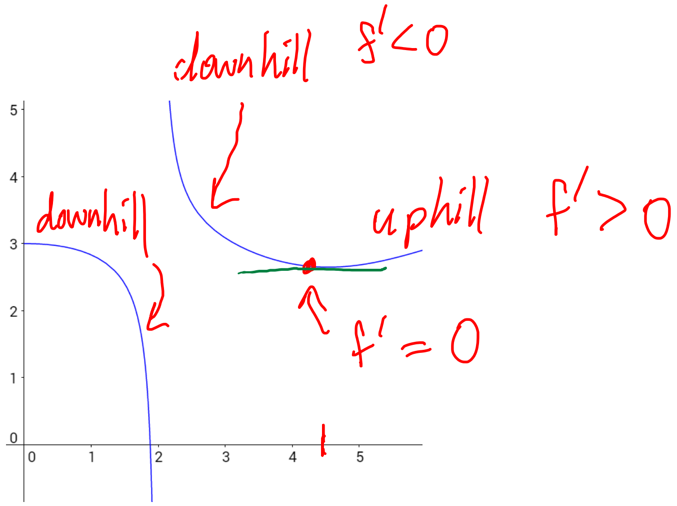
$= A_1 - A_2 = (1\cdot 2) + \frac{1}{2} 1\cdot 1$

$- \left[\frac{1}{2}\cdot 1\cdot 1 + 1\cdot 1\right]$

$= 2-1 = 1$

Boom
↓

25. Here is the graph of f:



downhill $f' < 0$

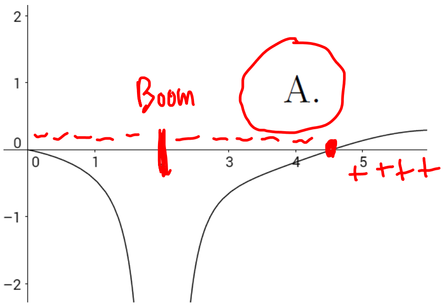
downhill

uphill $f' > 0$

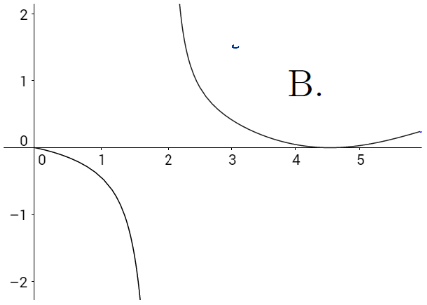
$f' = 0$

Find the graph of its derivative, f', below.

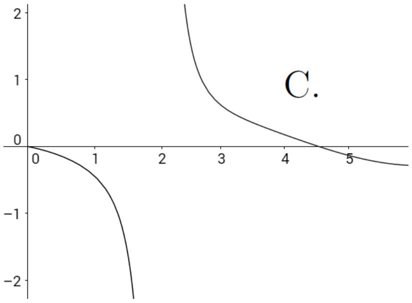
↓
Boom



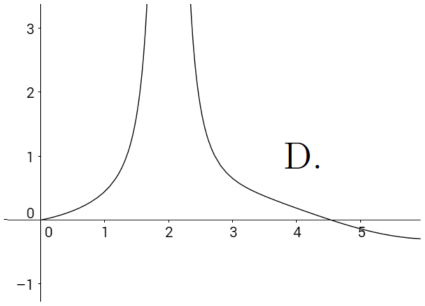
A.



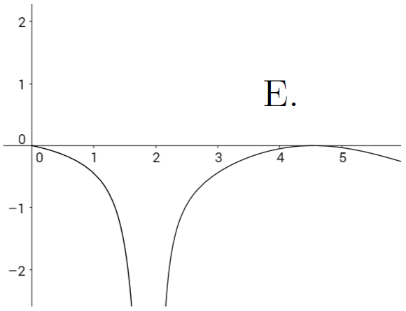
B.



C.



D.



E.