1. At time 0 a ball is thrown directly upward from a platform 10m tall. Its height above the ground after t seconds is $s = -5t^2 + 5t + 10$, where s is in meters. The ball hits the ground after 2 seconds. What is its velocity at impact?

$V = \frac{ds}{dt} = -10t + 5$

$t = 2: \quad V = -10 \cdot 2 + 5 = -15$

Note: If part of the problem were to find the moment of impact, you would solve $-5t^2 + 5t + 10 = 0$ to get $t = 2$.

A. 0

B. -5 m/s

C. -10 m/s

D. -15 m/s

E. -20 m/s

2. At which point(s) does the curve $y = x^3 - 6x^2 + 12x + 7$ have a horizontal tangent?

$\frac{dy}{dx} = 3x^2 - 12x + 12 = 0$

$3[x^2 - 4x + 4] = 0$

$3(x-2)^2 = 0$

$\frac{dy}{dx} = 0$ at $x = 2$.

A. $x = 0$ and $x = 1$

B. $x = 1$ and $x = 2$

C. $x = 0$ and $x = 2$

D. $x = 1$

E. $x = 2$

3. If $f(x) = \sqrt{x}\, e^{x-4}$, then $f'(4) =$

A. $\frac{9}{4}$

B. $\frac{1}{2}$

C. 0

D. $\frac{5}{4}$

E. $\frac{3}{4}$

$$f'(x) = \left(\tfrac{1}{2} x^{\frac{1}{2}-1}\right) e^{x-4} + \sqrt{x}\, e^{x-4} \cdot \frac{d}{dx}(x-4)$$

$$= \frac{e^{x-4}}{2\sqrt{x}} + \sqrt{x}\, e^{x-4}$$

$$f'(4) = \frac{e^0}{2 \cdot 2} + 2 \cdot e^0 = \frac{1}{4} + 2 = \frac{9}{4}$$

4. If $f(x) = (1 + \sin 2x)^{10}$, then $f'\left(\frac{\pi}{2}\right) =$

A. 1

B. 10

C. -10

D. 20

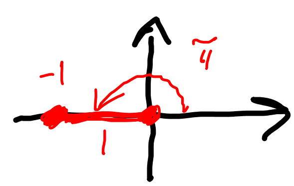
E. -20

$$f'(x) = 10(1 + \sin 2x)^9 \cdot [0 + (\cos 2x) \cdot 2]$$

$$= 20(1 + \sin 2x)^9 \cos 2x$$

$x = \frac{\pi}{2}$: $2x = \pi$. $\sin \pi = 0 = \frac{0}{1}$

$\cos \pi = -1 = \frac{-1}{1}$



$$f'\left(\tfrac{\pi}{2}\right) = 20(1+0)^9 \cdot (-1) = -20$$

5. If $g(x) = \tan\left(\frac{\pi}{2}\, f(x)\right)$, where $f(0) = 0$ and $f'(0) = 2$, then $g'(0) =$

A. 4
B. $\frac{\pi}{2}$
C. π
D. 2
E. Cannot be determined

$y = \tan u$ where $u = \frac{\pi}{2} f(x)$

$g'(x) = \frac{dy}{du} \cdot \frac{du}{dx} = (\sec^2 u) \cdot \left(\frac{\pi}{2} f'(x)\right)$

$= \left[\sec^2\left(\frac{\pi}{2} f(x)\right)\right] \cdot \left(\frac{\pi}{2} f'(x)\right)$

$g'(0) = \sec^2\left(\frac{\pi}{2} f(0)\right) \cdot \frac{\pi}{2} f'(0) = (\sec^2 0) \cdot \frac{\pi}{2} \cdot 2 = 1^2 \cdot \pi$

$= \pi$

6. If $f(x) = \ln\sqrt{\frac{x^3}{1-x^2}}$, then $f'(x) =$

A. $\frac{1}{2}\left(\frac{3}{x} - 1\right)$
B. $\frac{1}{2}\left(\frac{3}{x} + \frac{2x}{1-x^2}\right)$
C. $\frac{1}{2}\left(\frac{3}{x} - \frac{2x}{1-x^2}\right)$
D. $\frac{1}{2}\left(\frac{3}{x} + 1\right)$
E. $\frac{1}{2}\left(\frac{5}{x}\right)$

$= \ln\left(\frac{x^3}{1-x^2}\right)^{1/2} = \frac{1}{2} \ln \frac{x^3}{1-x^2}$

$f(x) = \frac{1}{2} \ln x^3 - \frac{1}{2} \ln(1-x^2)$

$f'(x) = \frac{1}{2} \cdot \frac{1}{x^3} \cdot 3x^2 - \frac{1}{2} \cdot \frac{1}{1-x^2} \cdot (-2x)$

$= \frac{3}{2} \cdot \frac{1}{x} + \frac{x}{1-x^2}$

7. Find an equation for the line tangent to the graph of $y = \dfrac{x^3}{\ln x}$ at the point (e, e^3).

$$\frac{dy}{dx} = \frac{(3x^2)\ln x - x^3 \cdot \frac{1}{x}}{(\ln x)^2}$$

A. $y = 2e^2x - e^3$

B. $y = 2e^2x + e^3 - 3$

C. $y = 2e^2x + e$

D. $y = -e^2x + e$

E. $y = -e^2x - e$

When $x = e$: $\dfrac{dy}{dx} = \dfrac{3e^2 \cdot 1 - e^2}{1^2} = 2e^2$

and $y = \dfrac{e^3}{\ln e} = e^3$

Tangent line: $\dfrac{y - e^3}{x - e} = 2e^2$ $\quad y = e^3 + 2e^2(x - e)$

$= 2e^2x - e^3$

8. Use implicit differentiation to find $\dfrac{dy}{dx}$ at the point $(1, 2)$ if $x^4 - 3x^2y + y^2 + y^3 = 7$.

$$4x^3 - [6xy + 3x^2y'] + 2yy' + 3y^2y' = 0$$

$$y'(-3x^2 + 2y + 3y^2) = -4x^3 + 6xy$$

$$y' = \frac{-4x^3 + 6xy}{-3x^2 + 2y + 3y^2}\Bigg|_{\substack{x=1\\y=2}} = \frac{-4 \cdot 1^3 + 6 \cdot 1 \cdot 2}{-3 \cdot 1^2 + 2 \cdot 2 + 3 \cdot 2^2}$$

$$= \frac{8}{-3+4+12} = \frac{8}{13}$$

A. $\dfrac{-2}{5}$

B. $\dfrac{1}{2}$

C. $\dfrac{4}{13}$

D. $-\dfrac{3}{5}$

E. $\dfrac{8}{13}$

9. Let $y = x^{\tan x}$. Find $\dfrac{dy}{dx}$.

A. $\dfrac{dy}{dx} = x^{\tan x}(\sec x \tan x \ln x + \tan x)$

B. $\dfrac{dy}{dx} = x^{\tan x} \quad \sec^2 x)\left(\dfrac{1}{x}\right)$

C. $\dfrac{dy}{dx} = x^{\tan x}\left(\sec^2 x \ln x + \dfrac{\tan x}{x}\right)$

D. $\dfrac{dy}{dx} = x^{\tan x - 1}(\tan x)$

E. $\dfrac{dy}{dx} = x^{\tan x} \quad \sec^2 x)$

$\operatorname{Ln} y = \operatorname{Ln} x^{\operatorname{Tan} x} = (\operatorname{Tan} x) \cdot \operatorname{Ln} x$

$\dfrac{d}{dx}(\operatorname{Ln} y) = (\operatorname{Sec}^2 x) \cdot \operatorname{Ln} x + (\operatorname{Tan} x) \cdot \dfrac{1}{x}$

$\dfrac{1}{y}\dfrac{dy}{dx} = \left[(\operatorname{Sec}^2 x) \cdot \operatorname{Ln} x + \dfrac{\operatorname{Tan} x}{x}\right]$

$\dfrac{dy}{dx} = y \cdot [\text{uuu}] = x^{\operatorname{Tan} x}\left[(\operatorname{Sec}^2 x)\operatorname{Ln} x + \dfrac{\operatorname{Tan} x}{x}\right]$

10. 60% of a radioactive substance decays in 3 hours. What is the half–life of the substance?

A. $3\left(\ln \dfrac{1}{5}\right)$ hours

B. $3\left(\dfrac{\ln \frac{1}{2}}{\ln \frac{2}{5}}\right)$ hours

C. $3\left(\dfrac{\ln \frac{1}{2}}{\ln \frac{5}{2}}\right)$ hours

D. $3\left(\dfrac{\ln \frac{5}{2}}{\ln \frac{1}{2}}\right)$ hours

E. $3\left(\dfrac{\ln \frac{2}{5}}{\ln \frac{1}{2}}\right)$ hours

60% gone after 3 hrs. So 40% remains.

$R = R_0 e^{-ct}$ ← R_0 present at time $t=0$. $(c>0)$

$\dfrac{40}{100} R_0 = R_0 e^{-c \cdot 3}$ ← 40% left after 3 hrs.

$\dfrac{2}{5} = e^{-3c}$

$\operatorname{Ln} \dfrac{2}{5} = -3c$

$c = -\dfrac{1}{3}\operatorname{Ln}\dfrac{2}{5} = \dfrac{1}{3}\operatorname{Ln}\dfrac{5}{2}$

$\boxed{\operatorname{Ln} a^{-1} = -\operatorname{Ln} a}$

Half life H:

$\dfrac{1}{2} R_0 = R_0 e^{-\left(\frac{1}{3}\operatorname{Ln}\frac{5}{2}\right)H}$

$\operatorname{Ln}\dfrac{1}{2} = -\left(\dfrac{1}{3}\operatorname{Ln}\dfrac{5}{2}\right)H$

$-\operatorname{Ln} 2 = -\dfrac{1}{3}\operatorname{Ln}\dfrac{5}{2} H$

$H = 3\dfrac{\operatorname{Ln} 2}{\operatorname{Ln}\frac{5}{2}} = \dfrac{3\left(-\operatorname{Ln}\frac{1}{2}\right)}{-\operatorname{Ln}\frac{2}{5}}$